6) Add the accrued interest [+] 156.6255

$$\#C = -\frac{\$189,157,370.92}{156.6255\%} \times \frac{11.6875}{8.8916} \times \frac{1.3298}{100,000} \times \frac{0.01\%}{0.01\%}$$

$$\#C = -2,111.00$$

4. How many contracts must you sell if you wish to decrease the duration of the portfolio owned to 5 years?

We can use the Hedge% formula to answer this question. We set the target duration to 5 and solve for the Hedge%:

$$\text{Hedge\%} = \frac{MD_U - MD_{Target}}{MD_U}$$

$$\text{Hedge\%} = \frac{11.6875 - 5}{11.6875} = 57.2193\%$$

We calculated above that we have to sell 2,111 March Treasury bond futures contracts to offset 100% of our underlying portfolio. We can therefore calculate how many contracts to sell in order to offset 57.2193%:

$$57.2193\% \times 2{,}111 = 1{,}207.90$$

We can therefore sell 1,208 contracts to reduce the duration of our portfolio to 5.

5. Please calculate the conversion factor for the following bond deliverable into the March 1994 Treasury bond futures contract:

Cash Market	
Issue:	Treasury
Maturity:	15-May-2018
Settlement:	16-Nov-1993
Coupon:	9 1/8
Market Price:	132 29/32
Face Value:	$100,000.00

Futures Market	
March 1993 Bund Futures Contract Price:	115 19/32
Settlement	16-Nov-1993
First Delivery Date:	1-Mar-1994
Days:	135

The most important thing to remember is to adjust the maturity of the bond by bringing it back from the actual maturity of 15 May 2018 to the nearest even calendar quarter end. For this bond, that means calculating with an adjusted maturity date of 1 March 2018:

Using the HP12C Calculator:

	Value	*Key*	*Display*
1) Clear financial registers and set accuracy to 4 digits		[f][REG] [f][4]	0.0000
2) Enter 8% YTM	8	[i]	8.0000
3) Enter the coupon	9.125	[PMT]	11.7500
4) Enter the delivery date for the March 1994 futures contract	3.011994	[ENTER]	3.0120
5) Enter the adjusted maturity date and calculate the bond's price	3.012018	[f][PRICE]	111.9223

6) Divide by 100	100	[÷]	1.1192

This is the conversion factor.

Using the HP19B Calculator:

	Value	*Key*	*Display*
1) Choose the financial menu		[FIN]	SELECT A MENU
2) Choose the bond menu		[BOND]	A/A SEMIANNUAL
3) Set the type of bond to		[TYPE]	
actual/actual semi-annual		[A/A]	
		[SEMI]	A/A SEMIANNUAL
4) Exit back to the bond menu		[EXIT]	A/A SEMIANNUAL
5) Enter the delivery date for the March 1994 futures contract	3.011994	[SETT]	SETT=03.01.1994 TUE
6) Enter the adjusted maturity date	3.012018	[MAT]	MAT=03.01.2018 THU
7) Enter the coupon	9.125	[CPN%]	CPN%=11.7500
8) Change menus		[MORE]	
9) Enter 8% YTM	8	[YLD%]	YLD%=8.0000
10) Solve for the price		[PRICE]	PRICE=111.9223
11) Divide by 100	100	[÷]	1.1192

Exhibit E

Relative Value Concepts

For Information contact:

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71 Murray Street, 10th fl.
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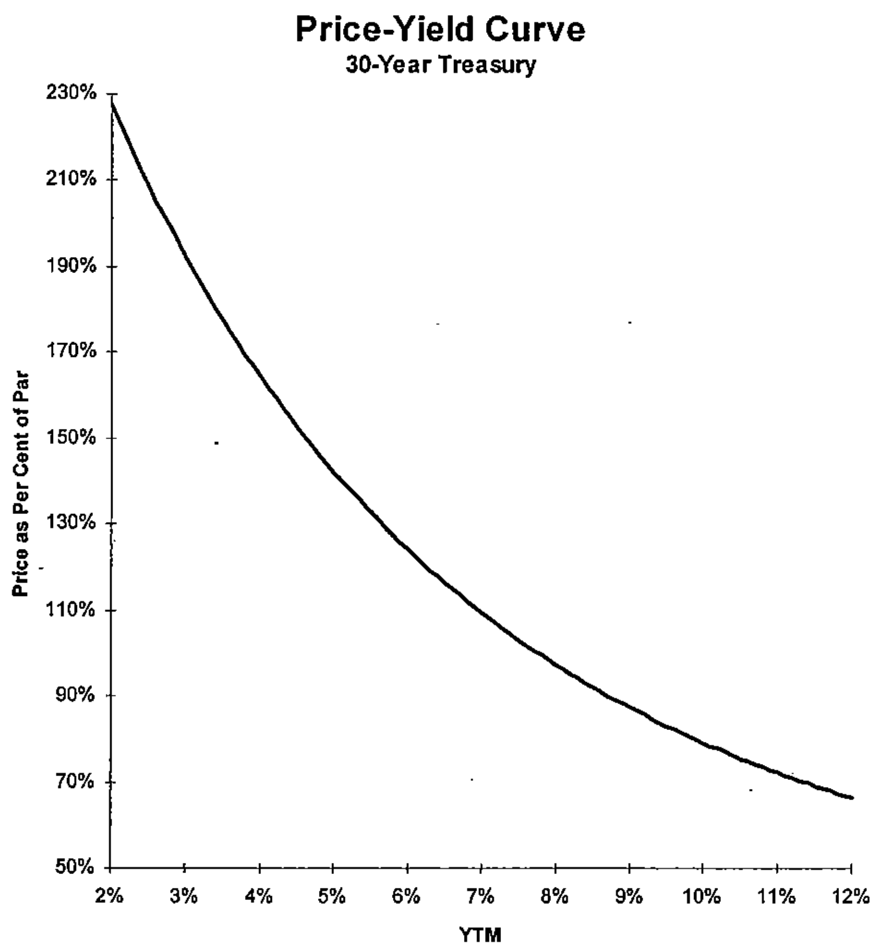
Relative Value Concepts

DURATION AND CONVEXITY

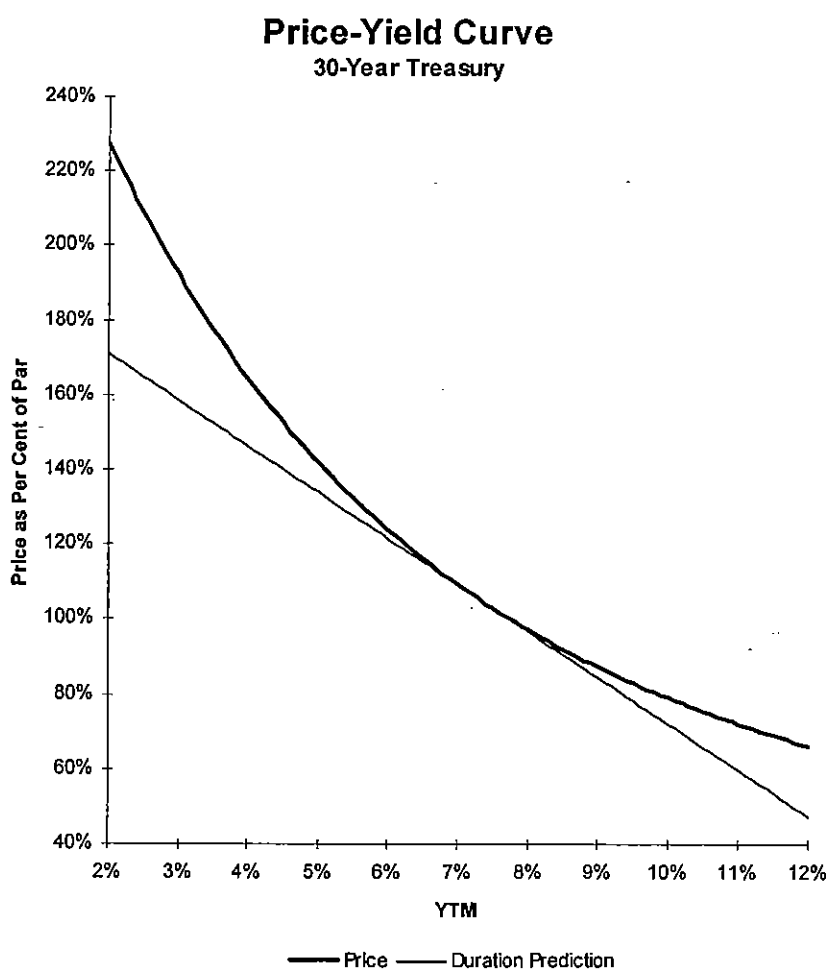
Below is a picture of the price-yield relationship for the U.S. Treasury long bond in early May 1995:



This bond has the following characteristics:

Settlement:	2-May-95
Issue:	15-Feb-95
Maturity:	15-Feb-25
Coupon:	7 5/8 %
Price:	103 11/32
Accrued:	1.6008
Dirty Price:	104.9446
YTM:	7.3456%

Modified duration does a reasonable job of describing price changes for small changes in the bond's YTM. For larger changes, however, it does not do a very good job:



Interestingly, modified duration is "wrong" in the same way whether rates move up or down. Its price prediction is **always below the true price.**

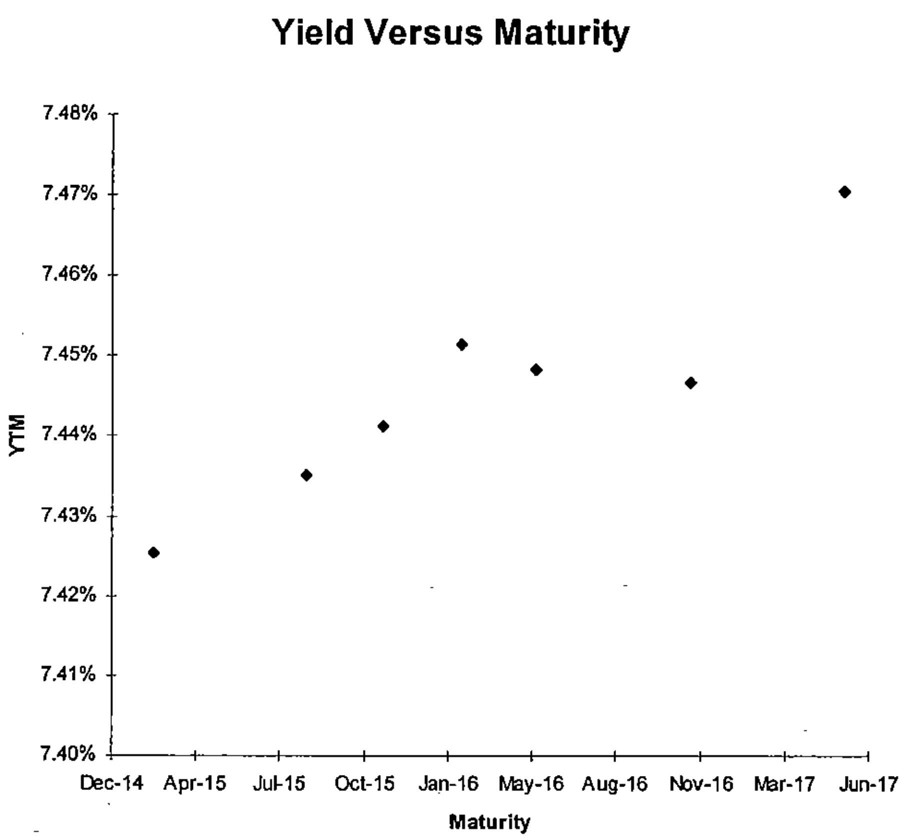
Risk Structure of Interest Rates

Bonds with greater duration have greater price risk. The yield curve compares yields on bonds with different maturities in order to obtain information about yield and maturity.

It is therefore reasonable to ask if there is a relationship between yield and price risk, or, more specifically, duration. The table below compares maturities and YTM for U.S. Treasuries having approximately 20 years remaining maturity in early May 1995.

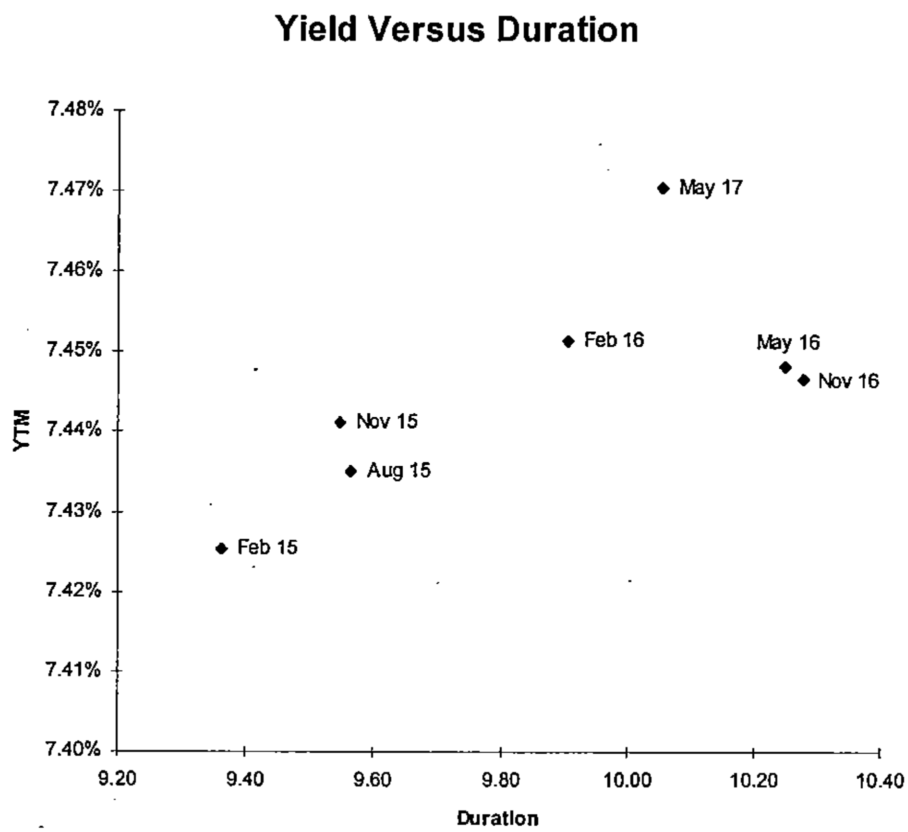
Maturity	Coupon	Price	YTM	Modified Duration
15-Feb-15	11 1/4 %	139 10/32	7.4255%	9.3628
15-Aug-15	10 5/8 %	133 4/32	7.4351%	9.5637
15-Nov-15	9 7/8 %	125 13/32	7.4412%	9.5491
15-Feb-16	9 1/4 %	118 27/32	7.4514%	9.9062
15-May-16	7 1/4 %	97 29/32	7.4482%	10.2487
15-Nov-16	7 1/2 %	100 18/32	7.4467%	10.2782
15-May-17	8 3/4 %	113 23/32	7.4706%	10.0543

We can look at this particular portion of the yield curve graphically in the usual way:



The yield on the bond maturing 15 February 2016 looks about a basis point too high, but otherwise the curve is fairly ordinary-looking.

We can also compare yield with duration:



Because its coupon is some 1.25% higher than the bonds maturing in May and November 2016, the May 2017 bond has a **lower** duration. By trading it at a slightly higher yield, the market would seem to be underpricing it slightly, given its lower risk.

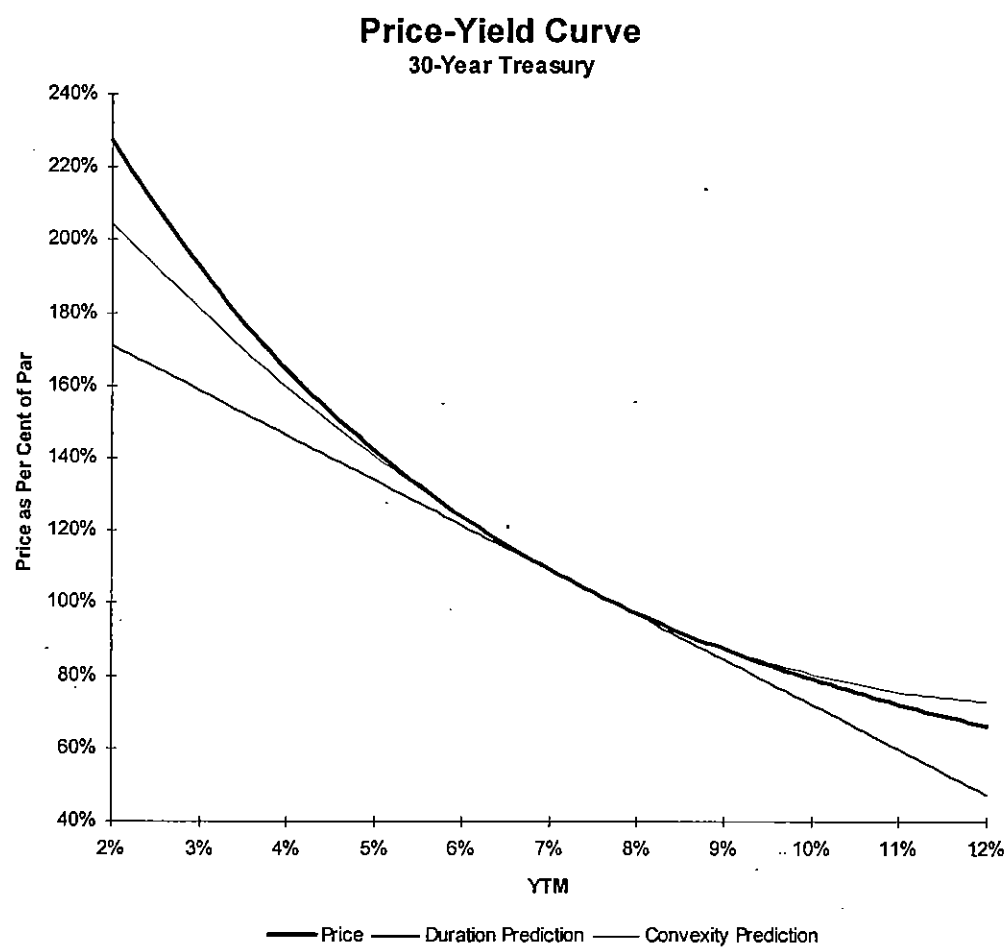
Bond Swaps

Bond swaps are sometimes structured based on this logic. for example, an investor might wish to increase yield while decreasing risk. In that case he could:

Action	Maturity	Coupon	Price	YTM	Modified Duration
Sell	15-Nov-16	7 1/2 %	100 18/32	7.4467%	10.2782
Buy	15-May-17	8 3/4 %	113 23/32	7.4706%	10.0543

Approximating the "True" Price-Yield Curve
In the graph above, we saw that duration does a good job of predicting a bond's price only for relatively small changes in yield.

To better approximate the price for larger changes in yield we need to use convexity:



As can be seen in the graph above, convexity is a desirable quality **for the owner of a bond.** The greater the convexity, the better the bond's performance when yields move lower.

The greater the convexity, the less risk of price drops when yields move higher.

DUMBBELL-BULLET ANALYSIS

A trading strategy employing views on the shape of the yield curve might be to trade two bonds — one short in maturity and the other long — against a third bond of medium maturity.

The combination of the two bonds is called a ***dumbbell*** (the name for a piece of weightlifting equipment consisting of a hand-held weight with a disk-shaped weight on each end).

The single bond is called a ***bullet***.

The goal of this trade is to create arbitrage profits without taking any risk. We are thus looking to take a long position in one and a short position in the other such that the two positions have equal duration risk and market values.

To illustrate this trading strategy, let us use the following three bonds (value date is 2 May 1995):

Bond	Maturity	Coupon	Price	Accrued	Dirty Price	YTM	Dollar Duration
A	15-Feb-00	7 1/8 %	100 29/32	1.4959	102.4021	6.8960%	4.0451
B	15-Feb-15	11 1/4 %	139 10/32	2.3619	141.6744	7.4255%	13.2629
C	15-Feb-05	7 1/2 %	102 31/32	1.5746	104.5433	7.0724%	7.1131

Bond A is a 5-year Treasury bond.

Bond B is a 20-year Treasury bond, one of those examined above.

Bond C is a 10-year Treasury bond.

The dumbbell will consist of a position in bonds A and B.

The bullet will be a position in C.

Dollar duration is calculated by multiplying the modified duration times the bond's dirty price. It represents the amount of absolute price change the bond will have for a 1% change in interest rates.

This can be shown by using Bond A.

Duration is calculated as follows. Each cash flow is priced, and then multiplied by time. In the exhibit below, PV factors are based on the bond's YTM, calculated using the A/A convention. Time is based on A/A days.

Date	Cash Flow	PV Factor	CF PV	Time	PV × T
2-May-95			102.3983%		4.091129
15-Aug-95	3.5625%	0.980390	3.4926%	0.287671	0.010047
15-Feb-96	3.5625%	0.947845	3.3767%	0.791781	0.026736
15-Aug-96	3.5625%	0.916253	3.2642%	1.290411	0.042121
17-Feb-97	3.5625%	0.885382	3.1542%	1.800000	0.056775
15-Aug-97	3.5625%	0.856193	3.0502%	2.290411	0.069862
16-Feb-98	3.5625%	0.827500	2.9480%	2.797260	0.082462
17-Aug-98	3.5625%	0.799769	2.8492%	3.295890	0.093906
15-Feb-99	3.5625%	0.773402	2.7552%	3.794521	0.104548
16-Aug-99	3.5625%	0.747484	2.6629%	4.293151	0.114323
15-Feb-00	103.5625%	0.722705	74.8452%	4.794521	3.588467

Duration is the sum of the cash flow PVs times time, divided by the bond's dirty price.

Modified duration is duration divided by 1 plus YTM:

$$\text{Modified Duration} = \frac{4.091129}{\left(1+\frac{6.8960\%}{2}\right)} = 3.95$$

Modified duration times the bond's dirty price gives us the dollar duration:

$$\text{Dollar Duration} = 3.95 \times 102.3983\% = 4.05$$

Dollar duration can be interpreted as the absolute amount of price change resulting from a 1% change in the bond's YTM.

We need to take a position in Bonds A and B which has the same market value as Bond C. That way, we can finance the trade.

At the same time we want the dollar duration of both positions to be the same. For small movements in interest rates, our two positions will have the same risk.

The first condition might be described as follows:

$$Z \times \left[X \times MV_A + (1-X) \times MV_B\right] = MV_C$$

where:

Z = The percent of face value of A and B needed to equal C
X = The percent of A needed
1–X = The percent of B needed
MV = The market value of each asset

The second condition can be described similarly as follows:

$$Z \times \left[X \times DD_A + (1-X) \times DD_B\right] = DD_C$$

where:

Z =	The percent of face value of A and B needed to equal C
X =	The percent of A needed
1–X =	The percent of B needed
DD =	The dollar duration of each asset

Using algebra, we can "solve" these two equations simultaneously to determine the exact amounts of A and B needed to exactly offset both the market value and dollar duration of C. Doing so we obtain the following equation for Z:

$$Z = \frac{DD_C - MV_C \times R}{DD_B - MV_B \times R}$$

$$\text{where: } R = \frac{DD_A - DD_B}{MV_A - MV_B}$$

The percent of Bond A can then be calculated as follows:

$$X = \frac{DD_B \times Z - DD_C}{DD_B \times Z - DD_A \times Z}$$

Using the above equations we can solve for the following amounts of A and B:

X	55.3570%
1–X	44.6430%
Z	87.1671%

The market value of the dumbbell is equal to the market value of Bond C:

$$87.1671\% \times [55.3570\% \times 102.4021 + 44.6430\% \times 141.6744] = 104.5433 = MV_C$$

The dollar duration of the dumbbell is also equal to that of Bond C:

$$87.1671\% \times [55.3570\% \times 4.0451 + 44.6430\% \times 13.2629] = 7.1131 = DD_C$$

The yield of the dumbbell can be calculated several different ways: weighted average yield, cash flow yield or horizon yield. For now we use the simple weighted average yield.

This is calculated using the respective market value percentages as weights:

$$55.3570\% \times 6.8960 + 44.6430\% \times 7.4255 = 7.1324\%$$

We can now describe our two possible positions more completely:

Bond	Maturity	Coupon	Price	Accrued	Dirty Price	YTM	Dollar Duration

C	15-Feb-05	7 1/2 %	102 31/32	1.5746	104.5433	7.0724%	7.1139
Dumbbell							
A	15-Feb-00	7 1/8 %	100 29/32	1.4959	49.4122	6.8960%	1.9521
B	15-Feb-15	11 1/4 %	139 10/32	2.3619	55.1311	7.4255%	5.1618
Total					104.5433	7.1324%	7.1139

Arbitrage Result Number 1

This yields a not-very-interesting result: It appears that we can grab a yield gain of only 0.06% by buying the Dumbbell and selling Bond C. The transaction is self-financing and of identical dollar duration. Intuitively, it seems reasonable that the yields should be about the same.

But is the yield difference calculated correctly?

Relative Value

In addition to weighted average yield, we can also compare the yields on the dumbbell and the bullet two other ways: cash flow yield and horizon yield.

Both of these are "better" ways of looking at the relative value of each strategy.

Cash Flow Yield

Calculating the simple average of the yields to maturity of the constituent bonds is a very rough approximation of the dumbbell's actual YTM.

This is because the two bonds have very different maturities.

YTM should be understood to be the single cash flow discounting rate which, when used to price each of the cash flows in the dumbbell, returns their aggregate market value.

There is no easy way to solve for this YTM. The easiest approach is to set it up on a spreadsheet and find the yield value through a solver function.

Solving for Cash Flow YTM Iteratively
The cash flows in the dumbbell are as follows:

Date	Cash Flow	PV Factor	CF PV	Time
2-May-95		1.000000	104.5433%	
15-Aug-95	3.9079%	0.979318	3.8271%	0.287671
15-Feb-96	3.9079%	0.945064	3.6932%	0.791781
15-Aug-96	3.9079%	0.911867	3.5635%	1.290411
17-Feb-97	3.9079%	0.879489	3.4370%	1.800000
15-Aug-97	3.9079%	0.848930	3.3176%	2.290411
16-Feb-98	3.9079%	0.818948	3.2004%	2.797260
17-Aug-98	3.9079%	0.790025	3.0874%	3.295890
15-Feb-99	3.9079%	0.762576	2.9801%	3.794521
16-Aug-99	3.9079%	0.735643	2.8748%	4.293151
15-Feb-00	52.1611%	0.709943	37.0314%	4.794521
15-Aug-00	2.1889%	0.685005	1.4994%	5.293151
15-Feb-01	2.1889%	0.660943	1.4467%	5.797260
15-Aug-01	2.1889%	0.637726	1.3959%	6.293151
15-Feb-02	2.1889%	0.615325	1.3469%	6.797260
15-Aug-02	2.1889%	0.593711	1.2996%	7.293151
17-Feb-03	2.1889%	0.572629	1.2534%	7.802740
15-Aug-03	2.1889%	0.552733	1.2099%	8.293151
16-Feb-04	2.1889%	0.533212	1.1672%	8.800000
16-Aug-04	2.1889%	0.514482	1.1262%	9.298630
15-Feb-05	2.1889%	0.496508	1.0868%	9.800000
15-Aug-05	2.1889%	0.479067	1.0486%	10.295890
15-Feb-06	2.1889%	0.462239	1.0118%	10.800000
15-Aug-06	2.1889%	0.446002	0.9763%	11.295890
15-Feb-07	2.1889%	0.430336	0.9420%	11.800000
15-Aug-07	2.1889%	0.415219	0.9089%	12.295890
15-Feb-08	2.1889%	0.400634	0.8770%	12.800000
15-Aug-08	2.1889%	0.386561	0.8461%	13.298630
16-Feb-09	2.1889%	0.372909	0.8163%	13.805479
17-Aug-09	2.1889%	0.359739	0.7874%	14.304110
15-Feb-10	2.1889%	0.347239	0.7601%	14.802740
16-Aug-10	2.1889%	0.334976	0.7332%	15.301370
15-Feb-11	2.1889%	0.323273	0.7076%	15.802740
15-Aug-11	2.1889%	0.311917	0.6828%	16.298630
15-Feb-12	2.1889%	0.300961	0.6588%	16.802740
15-Aug-12	2.1889%	0.290389	0.6356%	17.301370
15-Feb-13	2.1889%	0.280189	0.6133%	17.805479
15-Aug-13	2.1889%	0.270346	0.5918%	18.301370
17-Feb-14	2.1889%	0.260747	0.5708%	18.810959
15-Aug-14	2.1889%	0.251687	0.5509%	19.301370
16-Feb-15	41.1029%	0.242798	9.9797%	19.808219

3.9079% represents the total coupon of the dumbbell while both bonds are still outstanding:

$$87.1671\% \times \left(\frac{7.125\%}{2} \times 55.3570\% + \frac{11.25\%}{2} \times 44.6430\% \right) = 3.9079\%$$

Likewise, 2.1889% is the cash flow of the coupon on the longer Bond B.

The PV factors are based on the YTM we solve for. Each PV factor is calculated using YTM in the following relationship:

$$PVf_n = \frac{1}{\left[1+\frac{YTM}{2}\right]^{\left(\text{Periods}-1+\frac{\text{Accrual Days}}{\text{Actual Days in Accrual Period}}\right)}}$$

"Accrual days" refers to the number of days between settlement and the next semi-annual coupon date. "Actual days in accrual period" refers to the number of actual days in the 6-month period ending at the next semi-annual coupon date.

The solver function in Excel returns the following cash flow YTM: 7.2811%

This is the only value for cash flow YTM which yields the correct dumbbell PV of 104.5433%.

Approximate Cash Flow YTM

It is possible to closely approximate the cash flow yield in a portfolio by using the **dollar-duration-weighted average yield** instead of the simple weighted average yield.

The result is pretty good:

$$\frac{6.8960\% \times 1.9521 + 7.4255\% \times 5.1618}{1.9521 + 5.1618} = 7.2802\%$$

We can now describe our two possible positions more accurately:

Bond	Maturity	Coupon	Price	Accrued	Dirty Price	YTM	Dollar Duration
C	15-Feb-05	7 1/2 %	102 31/32	1.5746	104.5433	7.0724%	7.1139
Dumbbell							
A	15-Feb-00	7 1/8 %	100 29/32	1.4959	49.4122	6.8960%	1.9521
B	15-Feb-15	11 1/4 %	139 10/32	2.3619	55.1311	7.4255%	5.1618
Total					104.5433	7.2802%	7.1139

Arbitrage Result Number 2

This result is far more interesting: It appears that we can grab a yield gain of 0.21% by buying the Dumbbell and selling Bond C.

Is there any explanation for this? Does the relative riskiness of the dumbbell versus the bullet give us an explanation?

Relative Riskiness

To measure the relative riskiness of the dumbbell versus the bullet, we need to compute the **convexity** of both.

Dumbbell Convexity
Convexity is calculated as follows:

Dumbbell	YTM:	7.2811%			Convexity:	84.934235
Date	Cash Flow	PV Factor	CF PV	Time	PV × T	PV × T^2
2-May-95		1.000000	104.5433%			
15-Aug-95	3.9079%	0.979318	3.8271%	0.287671	0.011009	0.003167
15-Feb-96	3.9079%	0.945064	3.6932%	0.791781	0.029242	0.023154
15-Aug-96	3.9079%	0.911867	3.5635%	1.290411	0.045984	0.059338
17-Feb-97	3.9079%	0.879489	3.4370%	1.800000	0.061866	0.111358
15-Aug-97	3.9079%	0.848930	3.3176%	2.290411	0.075986	0.174039
16-Feb-98	3.9079%	0.818948	3.2004%	2.797260	0.089523	0.250420
17-Aug-98	3.9079%	0.790025	3.0874%	3.295890	0.101756	0.335377
15-Feb-99	3.9079%	0.762576	2.9801%	3.794521	0.113080	0.429085
16-Aug-99	3.9079%	0.735643	2.8748%	4.293151	0.123421	0.529866
15-Feb-00	52.1611%	0.709943	37.0314%	4.794521	1.775477	8.512561
15-Aug-00	2.1889%	0.685005	1.4994%	5.293151	0.079366	0.420098
15-Feb-01	2.1889%	0.660943	1.4467%	5.797260	0.083872	0.486225
15-Aug-01	2.1889%	0.637726	1.3959%	6.293151	0.087848	0.552839
15-Feb-02	2.1889%	0.615325	1.3469%	6.797260	0.091552	0.622301
15-Aug-02	2.1889%	0.593711	1.2996%	7.293151	0.094780	0.691247
17-Feb-03	2.1889%	0.572629	1.2534%	7.802740	0.097802	0.763125
15-Aug-03	2.1889%	0.552733	1.2099%	8.293151	0.100337	0.832113
16-Feb-04	2.1889%	0.533212	1.1672%	8.800000	0.102709	0.903843
16-Aug-04	2.1889%	0.514482	1.1262%	9.298630	0.104717	0.973724
15-Feb-05	2.1889%	0.496508	1.0868%	9.800000	0.106508	1.043774
15-Aug-05	2.1889%	0.479067	1.0486%	10.295890	0.107966	1.111609
15-Feb-06	2.1889%	0.462239	1.0118%	10.800000	0.109274	1.180164
15-Aug-06	2.1889%	0.446002	0.9763%	11.295890	0.110277	1.245678
15-Feb-07	2.1889%	0.430336	0.9420%	11.800000	0.111152	1.311593
15-Aug-07	2.1889%	0.415219	0.9089%	12.295890	0.111755	1.374123
15-Feb-08	2.1889%	0.400634	0.8770%	12.800000	0.112250	1.436798
15-Aug-08	2.1889%	0.386561	0.8461%	13.298630	0.112526	1.496442
16-Feb-09	2.1889%	0.372909	0.8163%	13.805479	0.112689	1.555727
17-Aug-09	2.1889%	0.359739	0.7874%	14.304110	0.112636	1.611152
15-Feb-10	2.1889%	0.347239	0.7601%	14.802740	0.112512	1.665486
16-Aug-10	2.1889%	0.334976	0.7332%	15.301370	0.112195	1.716730
15-Feb-11	2.1889%	0.323273	0.7076%	15.802740	0.111823	1.767104
15-Aug-11	2.1889%	0.311917	0.6828%	16.298630	0.111280	1.813718
15-Feb-12	2.1889%	0.300961	0.6588%	16.802740	0.110692	1.859936
15-Aug-12	2.1889%	0.290389	0.6356%	17.301370	0.109974	1.902695
15-Feb-13	2.1889%	0.280189	0.6133%	17.805479	0.109202	1.944401
15-Aug-13	2.1889%	0.270346	0.5918%	18.301370	0.108301	1.982056
17-Feb-14	2.1889%	0.260747	0.5708%	18.810959	0.107364	2.019618
15-Aug-14	2.1889%	0.251687	0.5509%	19.301370	0.106335	2.052417
16-Feb-15	41.1029%	0.242798	9.9797%	19.808219	1.976803	39.156944

The formula asks that we sum all of the products of PV × Time plus PV × Time^2, and then divide by $(1+\text{YTM}/2)^2$:

$$\text{Convexity} = \frac{\dfrac{\sum_{t=1}^{n}\left(t^2 \times PVCF_t + t \times PVCF_t\right)}{\text{Dirty Price}}}{\left(1+\dfrac{YTM}{2}\right)^2}$$

Dollar convexity is calculated by multiplying the convexity by the market value of the bond(s):

$$\text{Dollar Convexity} = 84.934 \times \frac{104.5433}{100} = 88.79$$

This can be understood to mean the pricing error resulting from using modified duration to predict the new price of the bullet. It is best to think of it as a relative term.

Bullet Convexity
For the bullet:

					Convexity:	63.505676
Date	**Cash Flow**	**PV Factor**	**CF PV**	**Time**	**PV × T**	**PV × T²**
2-May-95			104.5376%			
15-Aug-95	3.7500%	0.979899	3.6746%	0.287671	0.010571	0.003041
15-Feb-96	3.7500%	0.946570	3.5496%	0.791781	0.028105	0.022253
15-Aug-96	3.7500%	0.914241	3.4284%	1.290411	0.044240	0.057088
17-Feb-97	3.7500%	0.882677	3.3100%	1.800000	0.059581	0.107245
15-Aug-97	3.7500%	0.852857	3.1982%	2.290411	0.073252	0.167778
16-Feb-98	3.7500%	0.823571	3.0884%	2.797260	0.086390	0.241656
17-Aug-98	3.7500%	0.795290	2.9823%	3.295890	0.098295	0.323968
15-Feb-99	3.7500%	0.768422	2.8816%	3.794521	0.109342	0.414901
16-Aug-99	3.7500%	0.742035	2.7826%	4.293151	0.119463	0.512871
15-Feb-00	3.7500%	0.716829	2.6881%	4.794521	0.128882	0.617927
15-Aug-00	3.7500%	0.692346	2.5963%	5.293151	0.137426	0.727417
15-Feb-01	3.7500%	0.668700	2.5076%	5.797260	0.145374	0.842768
15-Aug-01	3.7500%	0.645861	2.4220%	6.293151	0.152419	0.959195
15-Feb-02	3.7500%	0.623802	2.3393%	6.797260	0.159006	1.080802
15-Aug-02	3.7500%	0.602497	2.2594%	7.293151	0.164779	1.201757
17-Feb-03	3.7500%	0.581696	2.1814%	7.802740	0.170206	1.328072
15-Aug-03	3.7500%	0.562044	2.1077%	8.293151	0.174792	1.449576
16-Feb-04	3.7500%	0.542744	2.0353%	8.800000	0.179106	1.576129
16-Aug-04	3.7500%	0.524207	1.9658%	9.298630	0.182790	1.699700
15-Feb-05	103.7500%	0.506401	52.5391%	9.800000	5.148828	50.458517

The dollar convexity for the bullet:

$$\text{Dollar Convexity} = 63.507 \times \frac{104.5433}{100} = 66.391$$

We can now describe our two possible positions more accurately:

Bond	Maturity	Coupon	Dirty Price	YTM	Dollar Duration	Dollar Convexity
C	15-Feb-05	7 1/2 %	104.5433	7.0724%	7.1139	66.39
Dumbbell						
A	15-Feb-00	7 1/8 %	49.4122	6.8960%	1.9521	
B	15-Feb-15	11 1/4 %	55.1311	7.4255%	5.1618	
Total			104.5433	7.2802%	7.1139	88.79

Arbitrage Result Number 3
This result is far more interesting: We cannot only take a yield gain of 0.21% by buying the Dumbbell and selling Bond C, but we get a position with far greater convexity, which, as we have seen, is a desirable property.

Something doesn't add up here!

Horizon Yield

Our analysis so far has paid no attention to the "expected developments" of yields over time. Specifically, we are treating each cash flow as if it can be reinvested at the various IRRs we are using.

Horizon yield is similar to realized compound yield in that it wants to treat the reinvestment of each cash flow at a market rate.

The simplest case is to use market PV factors to calculate cash flow reinvestment rates through a future date. This makes use of the terms structure of yields as it stands currently.

For example, suppose we wish to think in terms of a horizon of about nine months. For simplicity's sake, let us use the date of 15 February 1996, since we can readily calculate all the relevant PV factors. We will use the "generic" Treasury pricing curve calculated off the STRIPS market, as it reflects the correct zero-coupon Treasury rates.

To restate the yield curve as of 15 Feb 96 is quite simple. The logic is that we can invest today's PV of any future cash flow at the market rate existing today for maturity on 15 Feb 96.

For example, $1 invested today through 15 Feb 96 will be worth:

$$\frac{1}{0.952344} = 1.050041$$

$1 receivable on 15 August 1995 has a market PV today of 0.983125. Invested through 15 Feb 96 it will be worth:

$$\frac{0.983125}{0.952344} = 1.032322$$

Following the same logic, $1 receivable on 15 Feb 2000 is worth on 15 Feb 96:

$$\frac{0.722656}{0.952344} = 0.758819$$

We can thus calculate the PV as of 15 Feb 96 of every cash flow in both the dumbbell and the bullet:

Date	PV Factor	Bullet CF	Dumbbell CF
2-May-95	1.050041	109.6733%	109.8843%
15-Aug-95	1.032322	3.8712%	4.0342%
15-Feb-96	1.000000	3.7500%	3.9079%
15-Aug-96	0.970468	3.6393%	3.7925%
17-Feb-97	0.936833	3.5131%	3.6611%
15-Aug-97	0.903527	3.3882%	3.5309%
16-Feb-98	0.872847	3.2732%	3.4110%
17-Aug-98	0.842494	3.1594%	3.2924%
15-Feb-99	0.813782	3.0517%	3.1802%
16-Aug-99	0.786710	2.9502%	3.0744%
15-Feb-00	0.758819	2.8456%	39.5808%
15-Aug-00	0.731911	2.7447%	1.6021%
15-Feb-01	0.706809	2.6505%	1.5471%
15-Aug-01	0.681870	2.5570%	1.4926%
15-Feb-02	0.657588	2.4660%	1.4394%
15-Aug-02	0.633962	2.3774%	1.3877%
17-Feb-03	0.610500	2.2894%	1.3363%
15-Aug-03	0.588679	2.2075%	1.2886%
16-Feb-04	0.567022	2.1263%	1.2412%
16-Aug-04	0.547334	2.0525%	1.1981%
15-Feb-05	0.527810	54.7603%	1.1553%
15-Aug-05	0.502871		1.1007%
15-Feb-06	0.483183		1.0576%
15-Aug-06	0.465135		1.0181%
15-Feb-07	0.446267		0.9768%
15-Aug-07	0.429040		0.9391%
15-Feb-08	0.412305		0.9025%
15-Aug-08	0.396719		0.8684%
16-Feb-09	0.380640		0.8332%
17-Aug-09	0.366366		0.8019%
15-Feb-10	0.351928		0.7703%
16-Aug-10	0.338146		0.7402%
15-Feb-11	0.324856		0.7111%
15-Aug-11	0.312059		0.6831%
15-Feb-12	0.299918		0.6565%
15-Aug-12	0.288433		0.6314%
15-Feb-13	0.276948		0.6062%
15-Aug-13	0.266120		0.5825%
17-Feb-14	0.255947		0.5602%
15-Aug-14	0.246267		0.5391%
16-Feb-15	0.237244		9.7514%

The simple horizon yield through 15 Feb 96 is thus based on both strategies' market value today compared to their future values on the horizon date:

$$\text{Bullet} = \frac{109.6733\%}{104.5433\%} - 1 = 4.9127\% \qquad \text{Dumbbell} = \frac{109.8843\%}{104.5433\%} - 1 = 5.1088\%$$

Once again, the dumbbell proves to be the "richer" strategy.

HORIZON VALUE AND VOLATILITY

The resolution of this arbitrage lies in modeling future developments in the yield curve to incorporate interest rate volatility.

Specifically, we need to use a binomial tree to model either the short-term rate (a ***single factor model***) or the entire term structure of forward rates (a ***multiple factor model***).

Building binomial trees is beyond the scope of this module, but we can summarize the results, using a simplified single-factor tree and a simple dumbbell-bullet comparison[1].

Bond	Maturity	Coupon	Price	Per Cents	YTM	Duration	Convexity
C	20 years	10%	100		10%	8.5795	121.5637
Dumbbell							
A	10 years	10%	100	27.37%	10%	6.2311	
B	30 years	10%	100	72.63%	10%	9.4646	
Total			100		10%	8.5795	133.8108

The dollar duration weighted YTM of the dumbbell is identical to that of the bullet, indicating no arbitrage opportunity exists.

The horizon yield for both is also the same, 10% over a five-year horizon. This is because the yield curve is flat at 10%.

The convexities of the two positions are not the same, however. The greater convexity of the dumbbell appears attractive given that all else is equal.

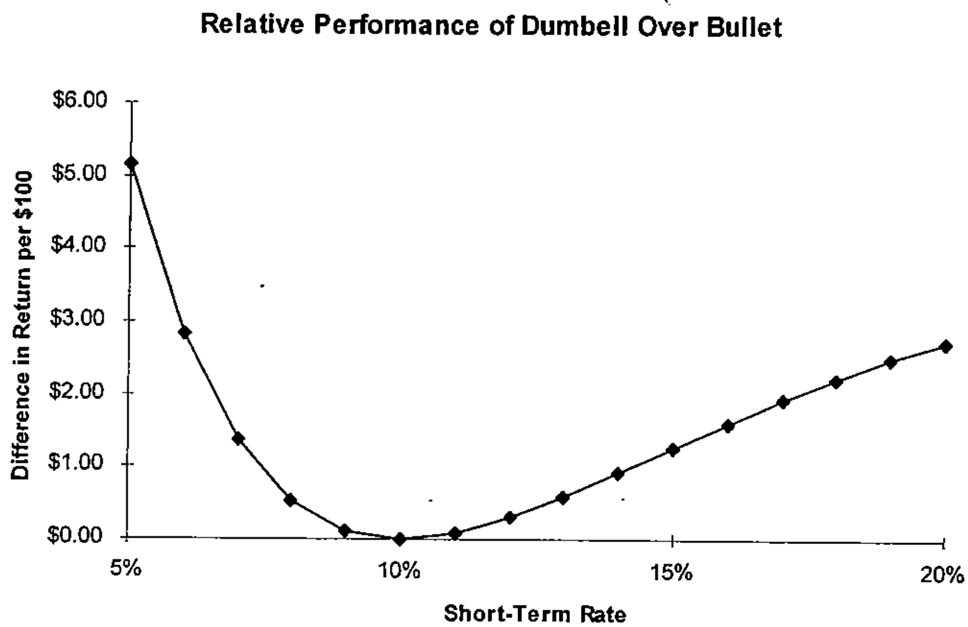
If we make some simplifying assumptions, we can test the two positions for assumed changes in the level of the 6-month rate over a 5-year horizon period:

1. Changes occur today and instantaneously affect both reinvestment rates (prior to 5 years) and horizon rates (after 5 years).

2. All instantaneous yield curve shifts occur in parallel fashion across the entire term structure.

[1] The following example is taken from *Active Total Return Management of Fixed Income Portfolios*, Dattatreya and Fabozzi; Probus Publishing, Chicago, 1989.

By comparing the horizon value of the two strategies to see which is worth more after five years, we can observe the effects of the convexity differences:



This suggests that no matter what happens, the added convexity of the dumbbell is worth owning.

But it does not tell the full story.

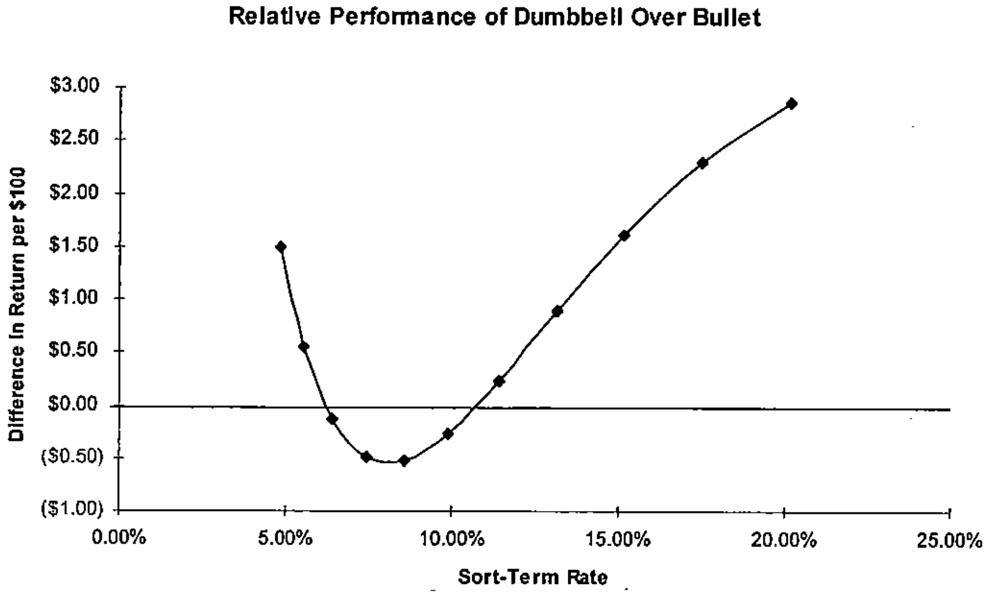
Our simple assumptions bear no resemblance to the way interest rates are likely to develop.

Rates do not instantaneously move by massive amounts and then hold still for 5 years.

Furthermore, we make no discrimination about how far rates are **likely** to move.

To incorporate a more realistic projection of horizon return, we need to use volatility and a binomial tree to measure the "likely" horizon values in terms of probability.

The graph which follows shows the excess value of the dumbbell strategy at various possible levels of the short-term rate 5 years from now. Clearly it is not always profitable to hold the dumbbell and sell the bullet.



Dattatreya and Fabozzi offer the following values for the horizon value of the two strategies, weighted by volatility and probability:

Short Rate	Probability	Bullet FV	Dumbbell FV	Difference
4.81%	0.1%	$208.36	$209.86	$1.50
5.56%	1.0%	$199.36	$199.92	$0.55
6.43%	4.4%	$190.07	$189.94	($0.13)
7.42%	11.7%	$180.60	$180.12	($0.48)
8.56%	20.5%	$171.11	$170.60	($0.51)
9.88%	24.6%	$161.79	$161.54	($0.26)
11.40%	20.5%	$152.84	$153.07	$0.24
13.15%	11.7%	$144.43	$145.32	$0.89
15.17%	4.4%	$136.76	$138.37	$1.61
17.49%	1.0%	$129.99	$132.28	$2.30
20.16%	0.1%	$124.25	$127.11	$2.87
Probability-Weighted Returns		**$162.24**	**$162.27**	**$0.03**

Finally, we can resolve this apparent market inefficiency. On a probability-weighted basis both strategies have the same expected value after 5 years.

Relative Value Concepts

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Exhibit G

Option Book Management

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Option Book Management

OVERVIEW

The task of managing books of swaps, caps, floors and swaptions has been raised to a science over the past few years, and primary research continues to bring new tools with which book managers do a better and better job.

Manager of swap books and the traders who work with them have to satisfy many different masters:

1. Clients and swap marketers whose deals must be priced and hedged.
2. Trading managers who want the book to be profitable.
3. Risk managers who want to know the *Value at Risk* of all the positions taken on the book and the return against the normal risk taken.
4. Senior managers who want no surprises in the form of losses due to "extraordinary" rate movements.

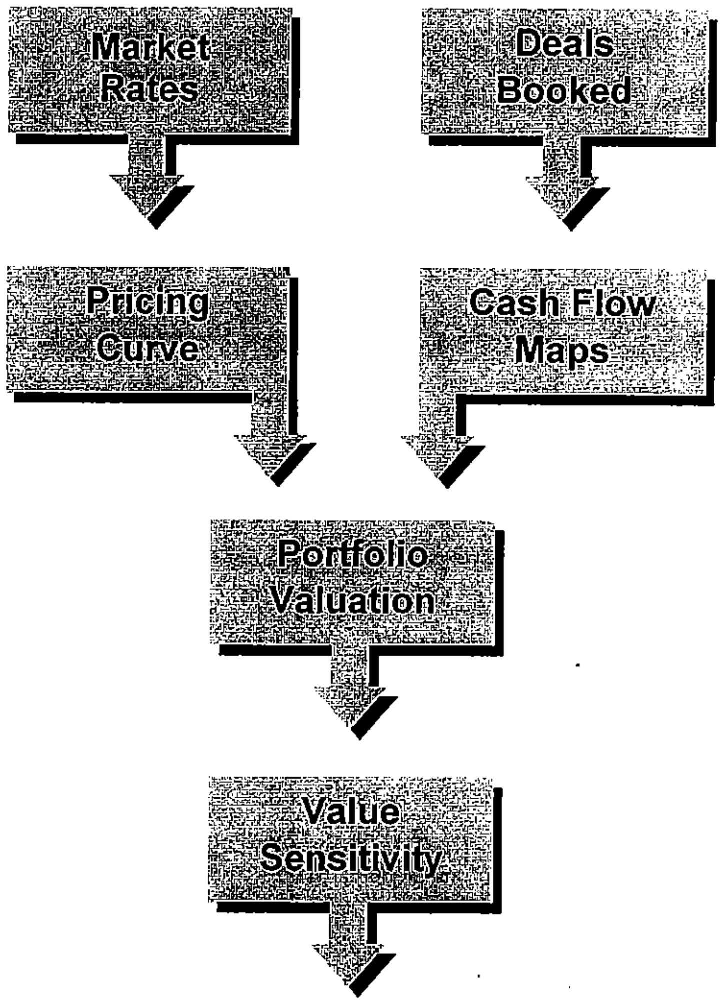
Systems which quantify trades and their changing values have become very sophisticated, and increasing efforts are underway to integrate trading ("front office"), record-keeping ("back office") and risk management ("middle office"). Systems sellers also come closer and closer to being able to accommodate new instruments without re-writing the entire system.

In this module we will examine the task of managing a book of interest rate options. We will describe this task as essentially that of a portfolio manager.

The value of the portfolio is subject to constant changes in interest rates and volatility, and the portfolio is changing constantly as new deals are done and old deals come off.

What drives the pricing decisions of the manager of such a portfolio? What leads him to prefer certain trades and run away from others? How does he hold this shifting bundle of cash flows together?

The management of interest rate options is similar to that of interest rate swaps with the addition of greater interest rate risk and volatility risk. The process is similar, too:



1. The first step is to identify the most liquid market instruments which can be used to manage the risk of the book. This is normally some combination of deposits, 3-month deposit futures and interest rate swaps.

2. Liquid market rates are used to calculate a pricing curve (sometimes called a *discount function*) which can be used to value everything on the book or proposed to the book.

3. Every deal booked is entered into a database which keeps track of the date and amount of every cash flow in every transaction on the book.

4. Instrument cash flows are mapped to the dates of the pricing curve. This is normally done using sophisticated interpolation methods which have the effect of splitting the actual instrument cash flows among the dates on the pricing curve.

5. Mapping the cash flows allows the book to be valued.

6. The book's value sensitivity to interest rate and volatility changes is measured and adjusted using liquid instruments if desired.

The focus of this module lies on the very last step: testing and hedging the book's sensitivity to changing interest rates and volatility levels. This task is the most difficult — and interesting — aspect of managing option books.

INTEREST RATE OPTION PRICING FUNDAMENTALS

Caps and Floors

Caps and floors are options on interest rate swap LIBOR resets.

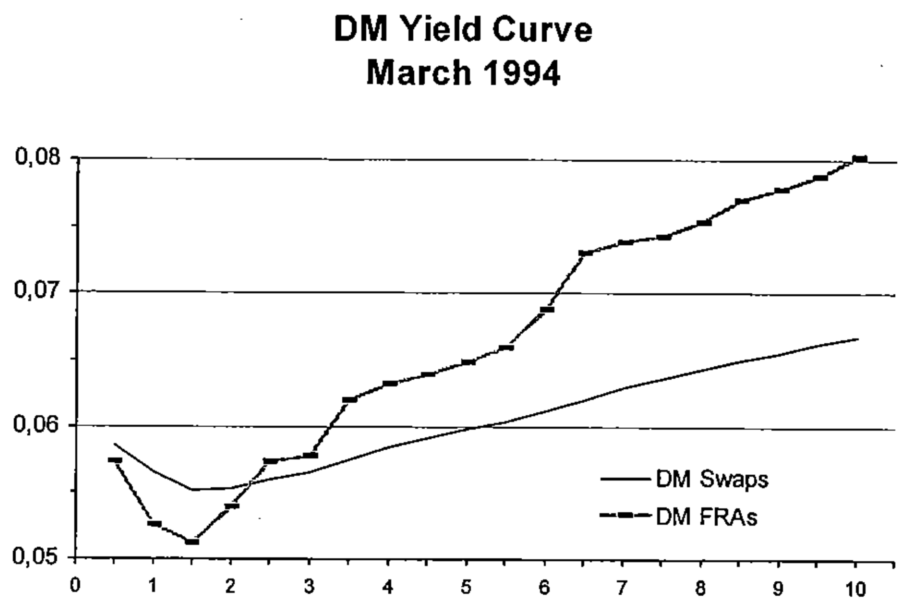
The money for a cap or floor is thus basically the swap rate — with several adjustments:

1. The main structural difference between swaps and caps and floors is that swaps almost always include the floating-rate for the period starting on the day the swap begins, while caps and floors normally do not.

2. In DM and many other currencies, swaps are quoted assuming the fixed rate payments are made once each year. Cap and floor strikes are quoted with the same compounding convention as the LIBOR rate underlying them: 6-month LIBOR is often the underlying rate.

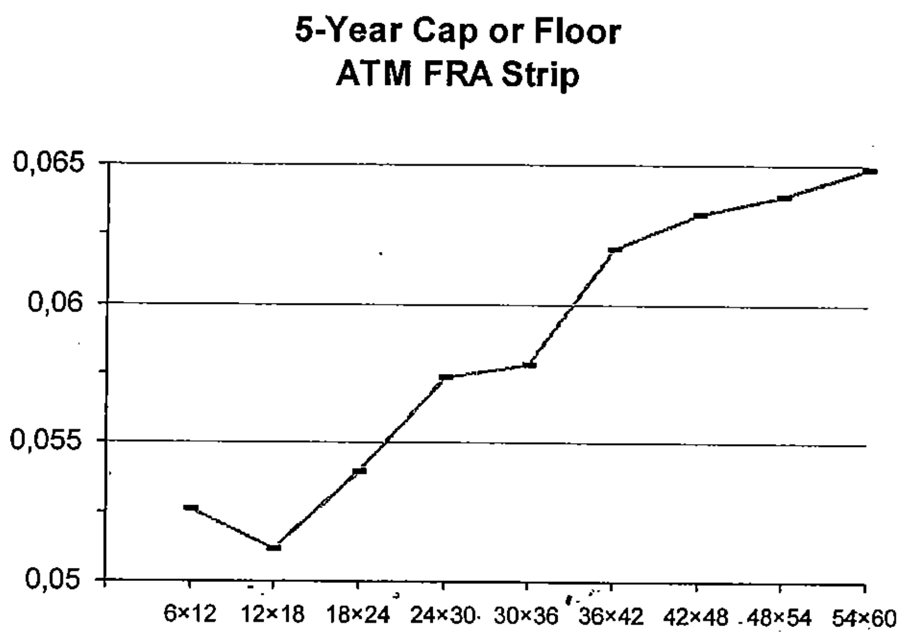
3. Finally, in order to be compared directly to LIBOR rates, which are quoted on a money market basis of actual days in a year of 360 days, cap and floor strikes are normally quoted according to the same convention.

In March 1994 the DM yield curve stood as shown below:



Just as the swap is the economic equivalent of the FRA strip, so, too, are the cap and floor the economic equivalent of options on the strip of successive FRAs.

For a 5-year cap or floor on 6-month LIBOR, therefore, we can say that the money for each successive FRA cap is the FRA itself:



Pricing a 5-Year Cap

To price a 5-year cap on 6-month LIBOR, we have to compare the strike rate of the cap to each of the successive FRAs in order to calculate the price of each FRA option.

The ATM cap rates are calculated in the following way:

$$\text{ATMCap} = \frac{\sum_{t=2}^{n} \text{FRA}_t \times \text{Days}_t \times \text{PVf}_t}{\sum_{t=2}^{n} \text{PVf}_t \times \text{Days}_t}$$

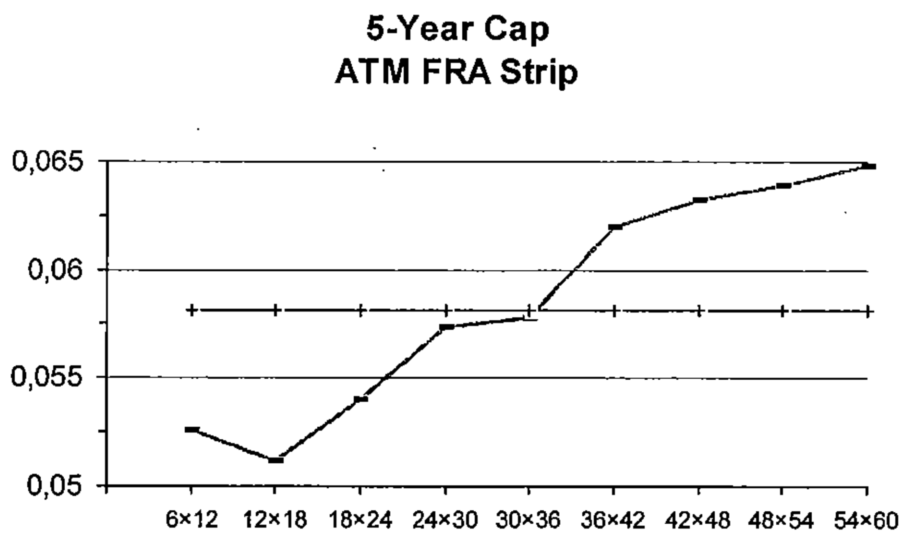
In other words, the ATM cap rate is equal to the PV of the floating-rate cash flows divided by the sum of the relevant PV factors weighted by the number of days in each 6-month period.

The formula can ignore the 360 above and below because it can be factored out of every term.

Using the formula above we can calculate the ATM cap strikes:

Date	Days	PVf's	Swaps	FRAs	ATM Caps & Floors	Swaps (A/360)
9-Mar-94		1,000000				
9-Sep-94	184	0,971537	5,8594%	5,7321%		
9-Mar-95	181	0,946505	5,6518%	5,2600%	5,2600%	5,5011%
9-Sep-95	184	0,922372	5,5145%	5,1191%	5,1899%	5,3763%
9-Mar-96	182	0,897858	5,5381%	5,4006%	5,2581%	5,3821%
9-Sep-96	184	0,872275	5,6007%	5,7382%	5,3740%	5,4499%
9-Mar-97	181	0,847633	5,6600%	5,7823%	5,4505%	5,5010%
9-Sep-97	184	0,821597	5,7497%	6,2000%	5,5674%	5,5930%
9-Mar-98	181	0,796284	5,8400%	6,3227%	5,6651%	5,6744%
9-Sep-98	184	0,771100	5,9089%	6,3900%	5,7471%	5,7452%
9-Mar-99	181	0,746745	5,9800%	6,4869%	5,8190%	5,8091%

For a 5-year cap struck at 5,81% (the ATM 5-year cap strike), we have the following picture:



Black-Scholes Pricing Model
We can price caps and floors on each of the successive FRAs using a modified version of the Black-Scholes option pricing model.

The Black-Scholes derived formula for a single-period cap (sometimes called a "caplet") is as follows:

$$\text{Cap Premium} = \text{Principal} \times PVf_{\text{Mat.}} \times \left[\text{FRA} \times N(d_1) - \text{Cap Strike} \times N(d_2)\right] \times \frac{\text{Days}}{360}$$

Where:

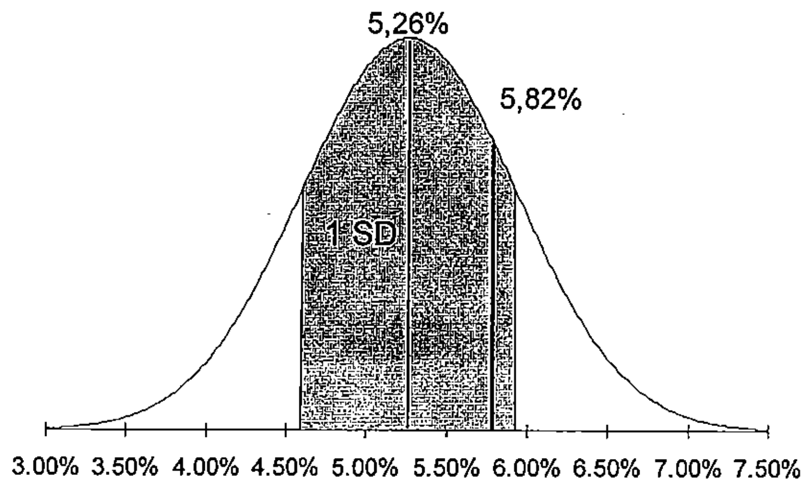
$$d_1 = \frac{\ln\left(\frac{\text{FRA}}{\text{Cap Strike}}\right) + \frac{\sigma^2 \times t}{2}}{\sigma \times \sqrt{t}}$$

$$d_2 = d_1 - \sigma \times \sqrt{t}$$

Principal =	Notional principal of the cap
$PVf_{\text{Mat.}}$ =	Present value factor through the end of the forward 3-month or 6-month period
FRA =	The market value of the floating rate for the relevant 3-month or 6-month period
N(...) =	The normal standard distribution function [NORMSDIST(...) in Excel]
ln(...) =	The natural logarithm function. Measures the percent difference between the FRA and the cap strike on a continuously compounded basis.
σ =	Annualized 3- or 6-month LIBOR volatility
t =	Time to expiry for each cap or floor expressed as a fraction of a year

Each FRA cap is priced separately. For each, we are pricing the likelihood that the market level of 6-month LIBOR exceeds the cap strike.

This might be pictured as follows:



In the picture above, we see the caplet for the 6×12 period:

- The strike is set at 5,82%
- The FRA (the "money") is 5,26%
- One standard deviation up and down at 18% volatility is 4,59% and 5,93%

The first part of the cap price formula above, FRA × N(d1), gives us the probability-weighted payoff of the cap, based on the shaded area to the right of the strike line.

The second part of the cap price formula, Strike × N(d2), gives us the probability that the cap will be exercised, based on the shaded area between the FRA and the strike.

In the same way, we can calculate the price of either a cap or a floor for each 6-month period.

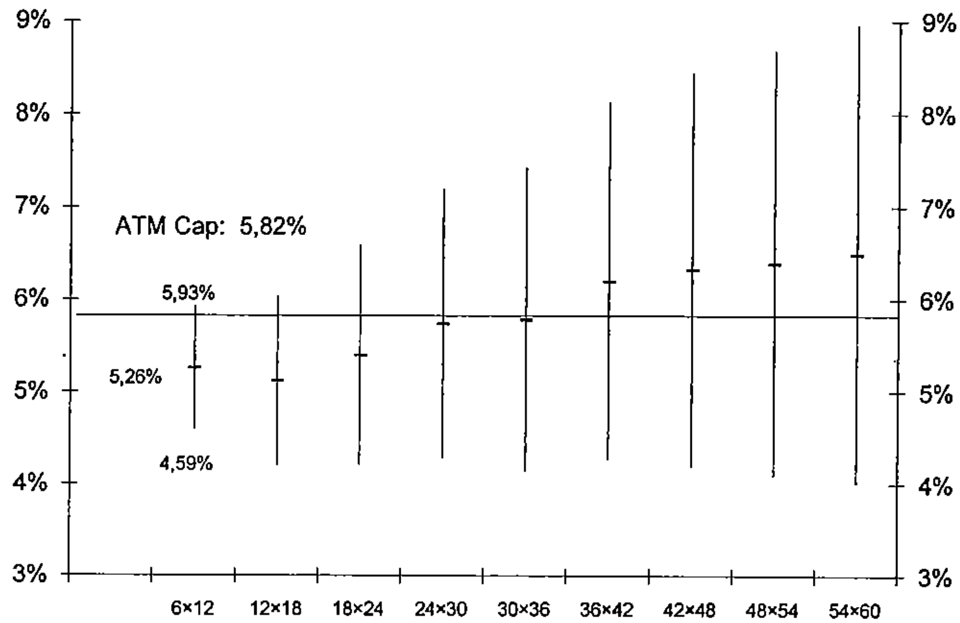
Doing so produces the following results:

Period	Days	Strike	FRA	PV Factor	Volatility	Years	Cap Premium	Floor Premium
0×6	184			0,971537				
6×12	181	5,8190%	5,2600%	0,946505	18,00%	0,504110	0,04%	0,31%
12×18	184	5,8190%	5,1191%	0,922372	18,00%	1,000000	0,06%	0,39%
18×24	182	5,8190%	5,4006%	0,897858	18,00%	1,502732	0,14%	0,33%
24×30	184	5,8190%	5,7382%	0,872275	18,00%	2,000000	0,24%	0,28%
30×36	181	5,8190%	5,7823%	0,847633	18,00%	2,504110	0,27%	0,29%
36×42	184	5,8190%	6,2000%	0,821597	18,00%	3,000000	0,40%	0,24%
42×48	181	5,8190%	6,3227%	0,796284	18,00%	3,504110	0,44%	0,23%
48×54	184	5,8190%	6,3900%	0,771100	18,00%	4,000000	0,47%	0,24%
54×60	181	5,8190%	6,4869%	0,746745	18,00%	4,504110	0,49%	0,24%
							2,5542%	2,5542%

The premiums for ATM caps and floors are equal.

These results make sense when compared to the relationship of the ATM cap and floor strike to the FRA strip:

- Where the cap strike is out of the money (i.e. above the FRA), the cap is relatively cheap.
- Where the cap strike is in the money, the cap is relatively expensive.
- The first two periods place the cap fairly far out of the money, so the first two "caplets" are relatively cheap.



Swaptions

An option on a swap is priced based on possible changes in future levels of LIBOR, just like a cap.

Unlike a cap, however, which offers multiple options on successive LIBOR resets, the swaption offers a single option on a whole series of LIBOR resets.

Because a swaption is an option to pay or receive LIBOR on a swap beginning in the future, the "money" for a swaption is the forward swap rate.

The volatility to be used is the amount of change in each of the FRAs across the strip over the life of the option, i.e. until the beginning of the forward period.

Since there is only one option, swaptions are normally cheaper than caps.

We can compare this by pricing a 3-year cap beginning in two years and comparing it to an option on a 3-year swap beginning in two years, a "2 into 3" swaption.

The price of the forward cap can be computed as above:

Period	Days	Strike	FRA	PV Factor	Volatility	Years	Cap Premium	Floor Premium
24×30	184	6,1382%	5,7382%	0,872275	18,74%	2,000000	0,20%	0,38%
30×36	181	6,1382%	5,7823%	0,847633	18,74%	2,504110	0,23%	0,38%
36×42	184	6,1382%	6,2000%	0,821597	18,98%	3,000000	0,35%	0,33%
42×48	181	6,1382%	6,3227%	0,796284	18,98%	3,504110	0,39%	0,32%
48×54	184	6,1382%	6,3900%	0,771100	15,54%	4,000000	0,36%	0,26%
54×60	181	6,1382%	6,4869%	0,746745	15,54%	4,504110	0,38%	0,25%
							1,9066%	1,9066%

The price of the swaption can be calculated using a similar modified Black-Scholes model:

$$\text{Swaption Premium} = \text{Prin.} \times \text{PVf}_{\text{Exp.}} \times \text{Mod.Dur.} \times \left[\text{Fwd} \times N(d_1) - \text{Strike} \times N(d_2)\right]$$

Where:

$$d_1 = \frac{\ln\left(\frac{\text{Fwd}}{\text{Strike}}\right) + \frac{\sigma^2 \times t}{2}}{\sigma \times \sqrt{t}}$$

$$d_2 = d_1 - \sigma \times \sqrt{t}$$

Principal = Notional principal of the swaption

PVf_{Exp} = Present value factor through the expiry of the swaption

Mod. Dur. = Modified duration of the forward swap (converts yield volatility to a price)

Fwd = The market value for the forward swap rate

N(...) = The normal standard distribution function [NORMSDIST(...) in Excel]

ln(...) = The natural logarithm function. Measures the percent difference between the Fwd and the swaption strike on a continuously compounded basis.

σ = Annualized swap rate volatility

t = Time to expiry of the swaption expressed as a fraction of a year

In this case the forward swap rate is 6,3210%.

The 2-year swaption volatility corresponding to the FRA volatilities above is 18,5%.

The ATM 2×3 swaption would cost 1,4549% of the notional principal.

This compares with the cap cost shown above of 1,9066%.

OPTION PRICE SENSITIVITIES

To get a feel for the sensitivity of interest rate option prices as a function of the relevant variables (interest rates, volatility and time), we will use a very simple single-period cap, an option on a single FRA.

Simple Single-Period Cap

The value of a cap is a function of several variables:

1. The level of the FRA
2. Volatility in the FRA
3. Time to maturity

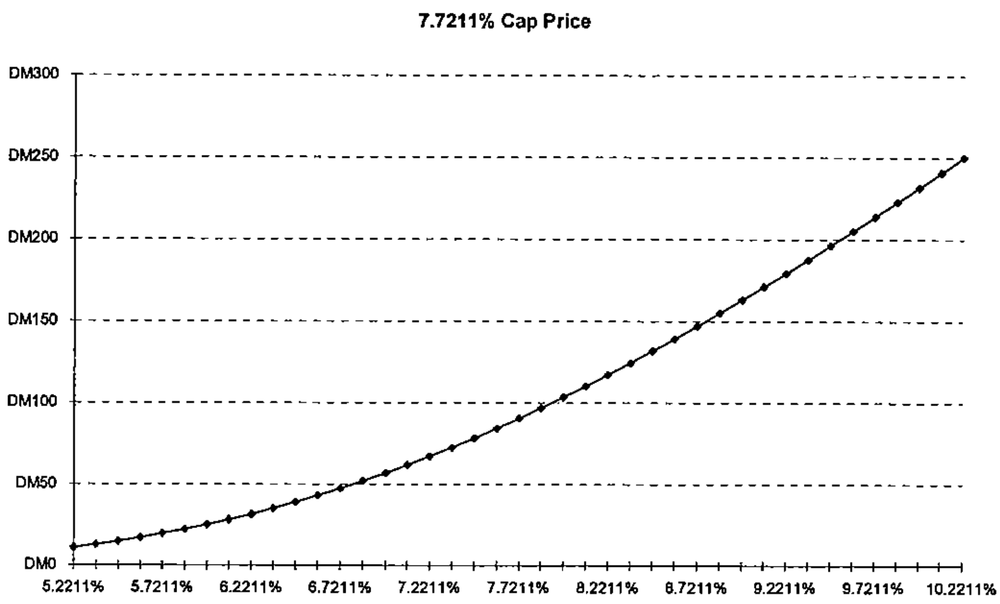
As each of these variables change, the cap price also changes. Cap price sensitivities measure the ratio of cap price change to changes in one of the variables. In the example following the sensitivities are all expressed in DM per one unit change in the variable.

Relevant Market Information

Position	Long
Face Value	DM25,000
Strike	7.7211%
FRA	7.7211%
Maturity	5 years
Days	184 days
PV Factor	0.703507
Volatility	15.50%
Time	4.4962
Value	DM90.5967

Delta

The primary determinant to the price of the option is the level of the underlying, in this case the FRA. This is evident on the following graph:



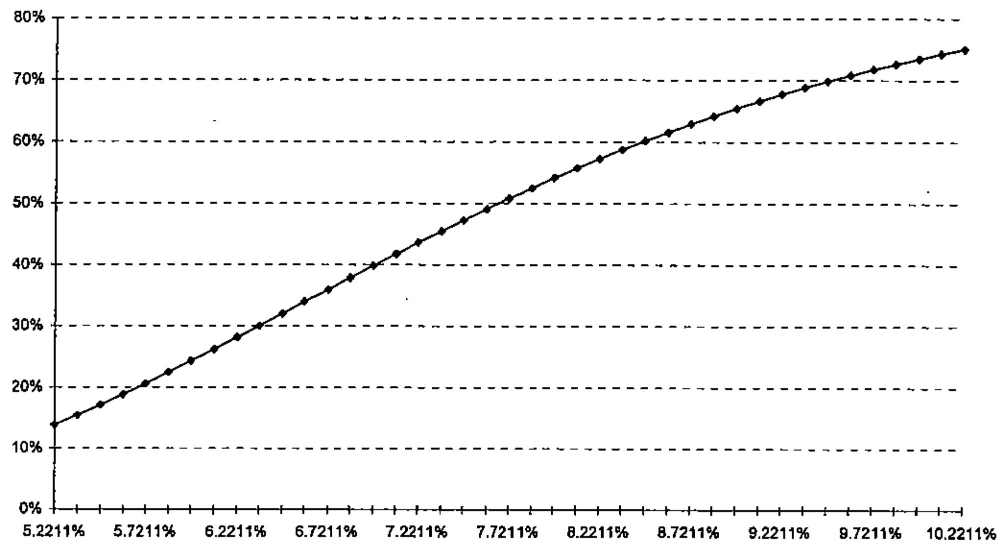
The sensitivity of the option premium with respect to movement in the underlying is known as the delta (Δ) of an option. Delta, from the Greek meaning change.

Deltas for caps are positive, for floors deltas are negative.

- **The equivalent FRA position needed to replicate the payoff of a cap is equal to the delta of the option.**

Deltas for caps vary between 0 and 1. For deep in the money options delta approaches 1 and for deep out of the money options delta approaches 0.

Deltas for floors vary between 0 and -1. For deep in the money options, delta approaches -1 and for deep out if the money options delta approaches 0.



Formula for Delta

For a caplet, delta for a 1 basis point change in the FRA may be calculated as follows:

$$\Delta = N(d_1) \times \text{Principal} \times \frac{\text{Days}}{360} \times \text{PVf}_{\text{Mat.}} \times 0.01\%$$

Where:
$$d_1 = \frac{\ln\left(\frac{\text{FRA}}{\text{Cap Strike}}\right) + \frac{\sigma^2 \times t}{2}}{\sigma \times \sqrt{t}}$$

N(...) = The normal standard distribution function [NORMSDIST(...) in Excel]

Principal = Notional principal of the cap

$\text{PVf}_{\text{Mat.}}$ = Present value factor through the end of the forward 3-month or 6-month period

FRA = The market value of the floating rate for the relevant 3-month or 6-month period

ln(...) = The natural logarithm function. Measures the percent difference between the FRA and the cap strike on a continuously compounded basis.

σ = Annualized 3- or 6-month LIBOR volatility

t = Time to expiry for each cap or floor expressed as a fraction of a year

Delta-Neutral Hedging
Delta neutral hedging is a strategy whereby changes in the option position due to a change in the underlying are offset by a change in the underlying position.

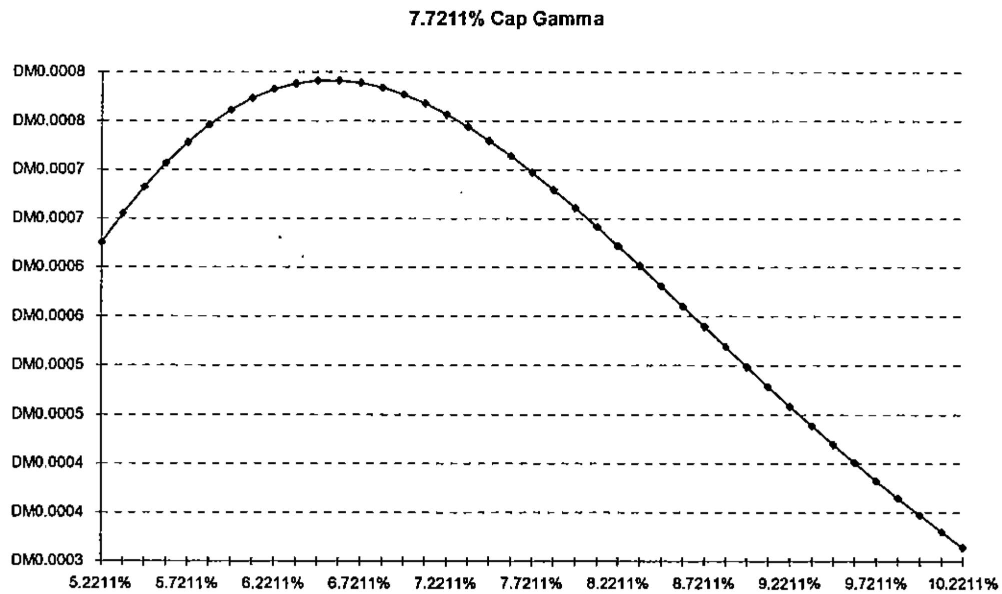
Gamma

As the FRA moves the delta changes. Delta is not a static measurement, but is dependent on the level of interest rates.

The gamma γ statistic measures the change in the delta, or the second order effect of the change in an option price due to a change in the underlying.

Gamma is greatest for at the money options.

The greatest change in delta occurs at the money. This is where the most uncertainty is, since the option can move in or out of the money with a slight shift in the underlying.



For the buyer of options — both caps and floors — gamma is always positive.

For caps, as the FRA increases, delta increases.

For floors, as the FRA increases, delta becomes less negative or more positive.

The price of a cap changes more rapidly for rising prices than the delta would suggest and falls less slowly for falling prices than the delta would suggest.

The price of a floor falls more slowly due to an increase in spot than delta would suggest and increases more rapidly due to a drop in the spot, than delta would suggest.

Formula for Gamma
For a caplet, gamma for a 1 basis point change in the FRA may be calculated as follows:

$$\Gamma = \frac{N'(d_1)}{FRA \times \sigma \times \sqrt{t}} \times \text{Principal} \times \frac{\text{Days}}{360} \times PVf_{Mat.} \times 0.01\%^2$$

Where: $$N'(d_1) = \frac{\exp\left(-\frac{(d_1^2)}{2}\right)}{\sqrt{2 \times \pi}}$$

Principal = Notional principal of the cap
$PVf_{Mat.}$ = Present value factor through the end of the forward 3-month or 6-month period
FRA = The market value of the floating rate for the relevant 3-month or 6-month period
exp(...) = The exponent function, also expressed as *e*.
σ = Annualized 3- or 6-month LIBOR volatility
t = Time to expiry for each cap or floor expressed as a fraction of a year

Gamma is valuable to option buyers. Gamma is dangerous for option sellers.

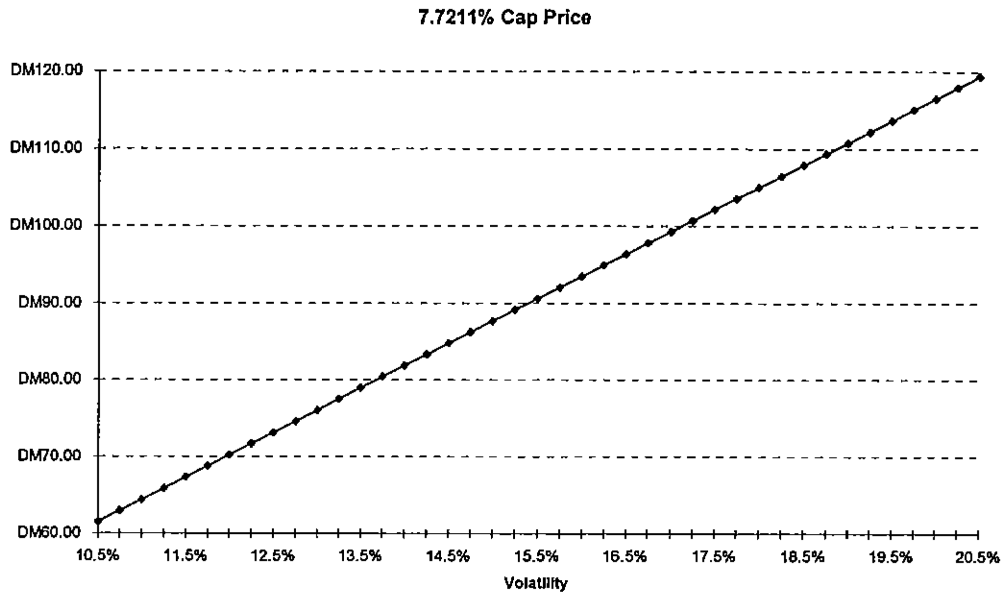
Delta-Neutral Hedging and Gamma
If gamma is small, delta only changes slowly and adjustments to keep a portfolio delta neutral are made relatively infrequently.

If gamma is large, delta is highly sensitive to a change in the underlying asset, and adjustments to keep the portfolio delta neutral have to be made fairly frequently.

Vega
Vega, also referred to as Kappa, is the rate of change of the option value with respect to a change in the volatility of the underlying.

If vega is high the option is sensitive to changes in volatility.

Vega is always positive for buyers of options and negative for sellers of options.



Vega is usually defined for a 1% increase in volatility.

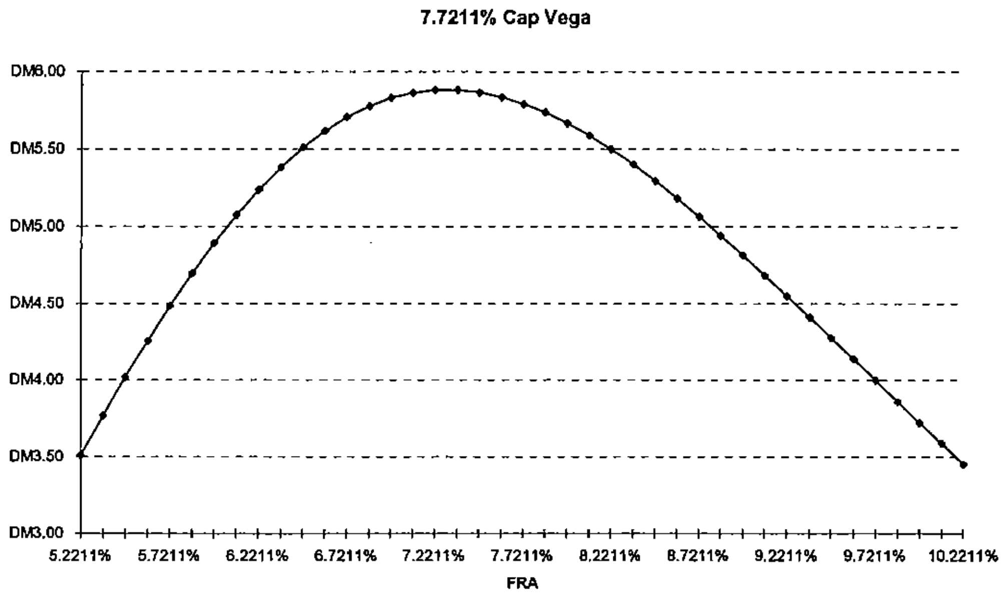
Formula for Vega
For a caplet, vega for a 1% change in volatility may be calculated as follows:

$$\text{Vega} = N'(d_1) \times \text{FRA} \times \sqrt{t} \times \text{Principal} \times \frac{\text{Days}}{360} \times \text{PVf}_{\text{Mat.}} \times 1\%$$

Where: $$N'(d_1) = \frac{\exp\left(-\frac{(d_1^2)}{2}\right)}{\sqrt{2 \times \pi}}$$

Principal = Notional principal of the cap
$\text{PVf}_{\text{Mat.}}$ = Present value factor through the end of the forward 3-month or 6-month period
FRA = The market value of the floating rate for the relevant 3-month or 6-month period
exp(...) = The exponent function, also expressed as *e*.
σ = Annualized 3- or 6-month LIBOR volatility
t = Time to expiry for each cap or floor expressed as a fraction of a year

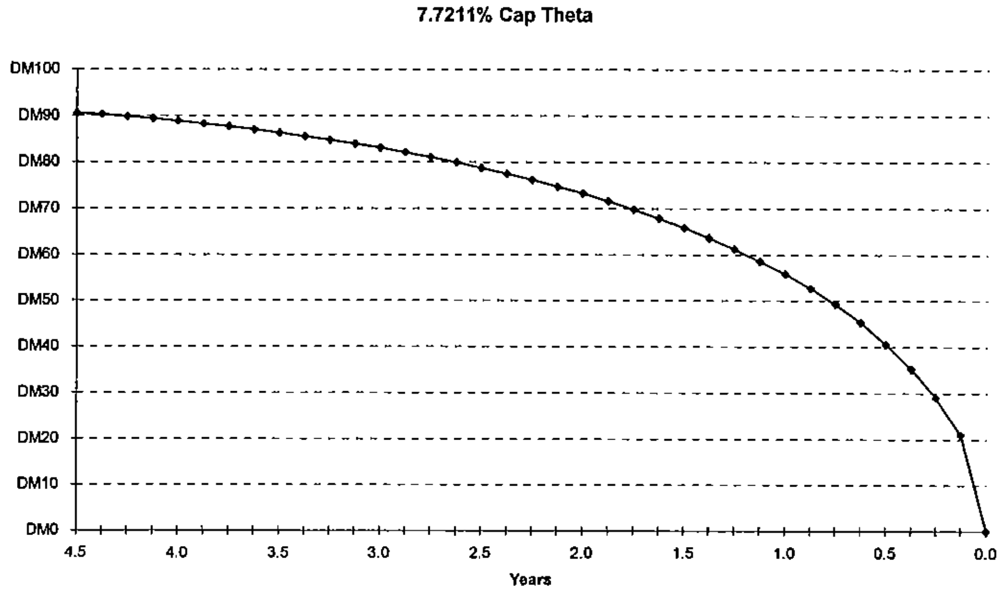
Vega is greatest for at the money options. The profile of vega vs. the FRA is similar to the gamma profile.



Theta

Theta describes changes in the value of the call as time passes. The primary relationship is that the value **decays** with each passing day.

A graph of the cap price relative to the time remaining to expiry shows the relationship very clearly:



Theta is the rate of change with respect to time of the option premium, with all else held constant.

Because an option is a decaying asset, theta will be negative most of the time.

OPTION PORTFOLIO ANALYSIS

Positions

Imagine you have the following positions on your (very simple) option book. All amounts are in thousands of DM:

Instrument	Position	Strike	Maturity	Face Value	Value
Cap #1	Short	7.50%	3	(DM15,000)	(DM141.60)
Cap #2	Long	8.50%	5	DM25,000	DM313.74
Floor #1	Long	6.50%	4	DM20,000	DM226.22
Floor #2	Short	6.25%	3	(DM50,000)	(DM325.05)
Pay Swaption	Short	7.50%	1×2	(DM25,000)	(DM180.57)
Rec. Swaption	Short	8.00%	2×3	(DM50,000)	(DM760.07)
Cash				DM1,000	DM1,000.00
TOTAL					**DM132.67**

An option pricing model tells you that the positions have the following sensitivities:

Instrument	Value	Δ	Γ	Vega	θ
Cap #1	(DM141.60)	(DM1.2197)	(DM0.0036)	(DM9.75)	DM0.14
Cap #2	DM313.74	DM2.5850	DM0.0074	DM34.07	(DM0.27)
Floor #1	DM226.22	(DM2.1518)	DM0.0075	DM19.02	(DM0.26)
Floor #2	(DM325.05)	DM3.6132	(DM0.0153)	(DM28.67)	DM0.55
Pay Swaption	(DM180.57)	(DM1.9387)	(DM0.0059)	(DM11.16)	DM0.26
Rec. Swaption	(DM760.07)	DM5.0098	(DM0.0121)	(DM44.47)	DM0.46
Cash	DM1,000.00	DM0.0000	DM0.0000	DM0.00	DM0.00
TOTAL	**DM132.67**	**DM5.90**	**(DM0.02)**	**(DM40.97)**	**DM0.88**

The delta of the above position is not great. It tells us that the portfolio will lose about 5% of its value for each basis point move down in the yield curve. Clearly for large moves of interest rates this is risky.

Gamma is also working against us. As rates move lower, the loss from the delta will grow faster and faster. It will accelerate at DM0.02 per 0.01% of change in rates.

The vega position exposes us to changing volatility. We are short volatility, so any increase in volatility will cost us money at a rate of DM40.97 per 1% change in volatility.

Fionally, since we have mostly sold options, our liability for them decays at about DM0.88 per day assuming nothing else changes.

Mapping Cash Flows

The process of mapping the portfolio allows us to measure the exposure we have to changes in the underlying market input rates.

Liquid Market Rates

Exchange-traded money market futures (contracts on 3-month LIBOR such as the Eurodollar, Euromark, etc.) are the most liquid market instruments the trader can use to adjust the value sensitivity of the portfolio.

The DM yield curve can thus be defined by using the first 6 contracts of Euromark futures traded on LIFFE followed by the interest rate swaps out to 10 years.

To manage a portfolio, it is vital to know the sensitivity of the portfolio to movements in the liquid instruments which can be used to adjust it. To change the rate sensitivity, the book manager must trade, and he will always prefer to trade the most liquid instruments. Therefore, we want to price and measure sensitivity in terms of the same liquid instruments.

What is required is a table of sensitivities:

Instrument	Interest Rate Portfolio Sensitivity
Mar 95 Future	?
Jun 95 Future	?
Sep 95 Future	?
Dec 95 Future	?
Mar 96 Future	?
Jun 96 Future	?
2-Year Swap Rate	?
3-Year Swap Rate	?
4-Year Swap Rate	?
5-Year Swap Rate	?

If the above table is defined, a position in the most liquid instrument can be used to offset the risk.

As such the trader can pick and choose at what points along the yield curve to remove exposure by trading in those particular instruments.

In addition, new customer business (swaps, options, etc.) can be analyzed by the trader and the effect on the risk of the current book can be easily identified.

A trader has two ways to manage the position and risk of the portfolio:

1. Use exchange traded futures or interbank derivatives (swaps, caps, floors, etc.)
2. Execute customer transactions

Exchange traded futures and interbank trading both have a cost: For futures it is the commission cost and margin calls. For trading in the interbank market, the cost is the bid/offer spread.

If the trader can execute a deal with a customer that moves the risk of the book in a desirable direction the pricing may be slightly better (for the book) than the market quote.

The trader can afford to give better pricing because the alternative is to execute a transaction to hedge certain exposures which will cost money. Some of that savings can be passed onto the customer.

If a customer trade would change the sensitivity of the portfolio to an undesirable level, the trader may quote a price worse than the market.

If the customer trade is executed, the trader will have to go out and pay to execute transactions which will return the book to the desirable position.

Delta

We can delta hedge the portfolio by taking positions to offset the portfolio's base exposure to 0.01% changes in the market input rates. First we have to measure it.

Following is the change in the price of the first cap when each of the input rates is changed by 1 basis point up and down.

Cap #1 Input Rates		Rates Up 0.01%	Rates Down 0.01%	Average Change
Mar-95	94.83	DM0.18	(DM0.18)	DM0.18
Jun-95	94.49	DM0.16	(DM0.16)	DM0.16
Sep-95	94.10	DM0.15	(DM0.16)	DM0.16
Dec-95	93.70	DM0.11	(DM0.11)	DM0.11
Mar-96	93.36	DM0.05	(DM0.05)	DM0.05
Jun-96	93.08	DM0.03	(DM0.03)	DM0.03
2 Yr	6.4450%	DM0.28	(DM0.32)	DM0.30
3 Yr	6.8450%	(DM2.16)	DM2.13	(DM2.15)
4 Yr	7.0650%	DM0.00	DM0.00	DM0.00
5 Yr	7.2150%	DM0.00	DM0.00	DM0.00

If we measure the sensitivity of each instrument to 0.01% changes in the input rates, we obtain the following table of sensitivities for our portfolio:

Input Rates		Portfolio Sensitivities
Mar-95	94.83	DM0.1738
Jun-95	94.49	DM0.3972
Sep-95	94.10	DM0.5664
Dec-95	93.70	DM0.3128
Mar-96	93.36	DM0.0267
Jun-96	93.08	DM0.0147
2 Yr	6.4450%	(DM4.1043)
3 Yr	6.8450%	(DM2.7138)
4 Yr	7.0650%	(DM1.9659)
5 Yr	7.2150%	DM13.5947

These represent the delta of the portfolio to 0.01% changes in the market rates used to construct the pricing curve. Positive numbers mean a gain if the market rate rises.

If the price of the March 1995 futures contract moves down by 1 tick, i.e. if the rate implied by it (5.17%) moves up by 0.01%, the portfolio will gain DM0.1738.

Most of the delta exposure is to the 5-year swap rate. If it moves up by 0.01%, the portfolio will gain DM13.5947.

To "delta hedge" the portfolio, we need to know the sensitivities of each of the underlying input instruments to a one basis point change, too.

The futures are easy. Each basis point is worth the tick value, which is DM25. For the par swaps we measure the sensitivity by revaluing each par swap for a 0.01% change in the par swap rates. The numbers in both cases assume a unit value of DM1 mio.

Input Rates		Market Equivalents
Mar-95	94.83	(DM25)
Jun-95	94.49	(DM25)
Sep-95	94.10	(DM25)
Dec-95	93.70	(DM25)
Mar-96	93.36	(DM25)
Jun-96	93.08	(DM25)
2 Yr	6.4450%	(DM182.77)
3 Yr	6.8450%	(DM264.65)
4 Yr	7.0650%	(DM340.59)
5 Yr	7.2150%	(DM410.94)

Both futures and swaps lose value as rates rise. For the futures, this means that the equivalent position is long a futures contract. This position loses value as rates rise.

For the swaps, this is equivalent to receiving fixed. The receiver of fixed is similar to someone long a cash bond: as rates rise the position loses value.

By comparing the basis point sensitivity of the portfolio to that of the input instruments, we can describe how to delta hedge the portfolio:

Input Rates		Portfolio Sensitivities	Market Equivalents	Delta Hedge
Mar-95	94.83	DM0.1738	(DM25)	7.0
Jun-95	94.49	DM0.3972	(DM25)	15.9
Sep-95	94.10	DM0.5664	(DM25)	22.7
Dec-95	93.70	DM0.3128	(DM25)	12.5
Mar-96	93.36	DM0.0267	(DM25)	1.1
Jun-96	93.08	DM0.0147	(DM25)	0.6
2 Yr	6.4450%	(DM4.1043)	(DM182.77)	Pay(DM22.457)
3 Yr	6:8450%	(DM2.7138)	(DM264.65)	Pay(DM10.254)
4 Yr	7.0650%	(DM1.9659)	(DM340.59)	Pay(DM5.772)
5 Yr	7.2150%	DM13.5947	(DM410.94)	RecDM33.082

By buying the indicated number of futures contracts and either receiving or (paying) fixed in the indicated amounts (in DM mio) we can delta hedge the portfolio.

The exposures to the futures contracts will lose money if rates fall (if prices rise). To hedge this we need long positions in the futures.

The calculation of the 5-year par swap hedge is as follows:

$$\frac{13.5947 \times 1{,}000 \times 1{,}000{,}000}{410.94} = 33{,}082{,}000$$

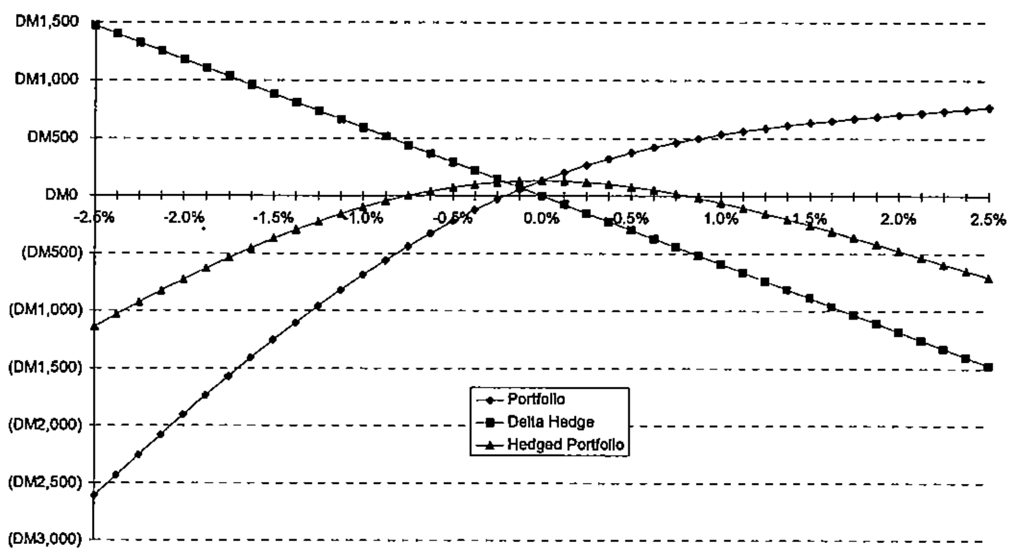
To hedge the 5-year par swap exposure, we need to receive fixed on a 5-year par swap of face value DM33.082 mio.

The 5-year swap equivalent risk is due to the 2×3 receiver swaption we sold and the 5-year cap #2 we own, which are both tied to changes in the 5-year par swap rate.

Gamma

The delta hedge is only valid for small changes in the yield curve, as can be seen in the following graph.

Portfolio Value As The Yield Curve Changes

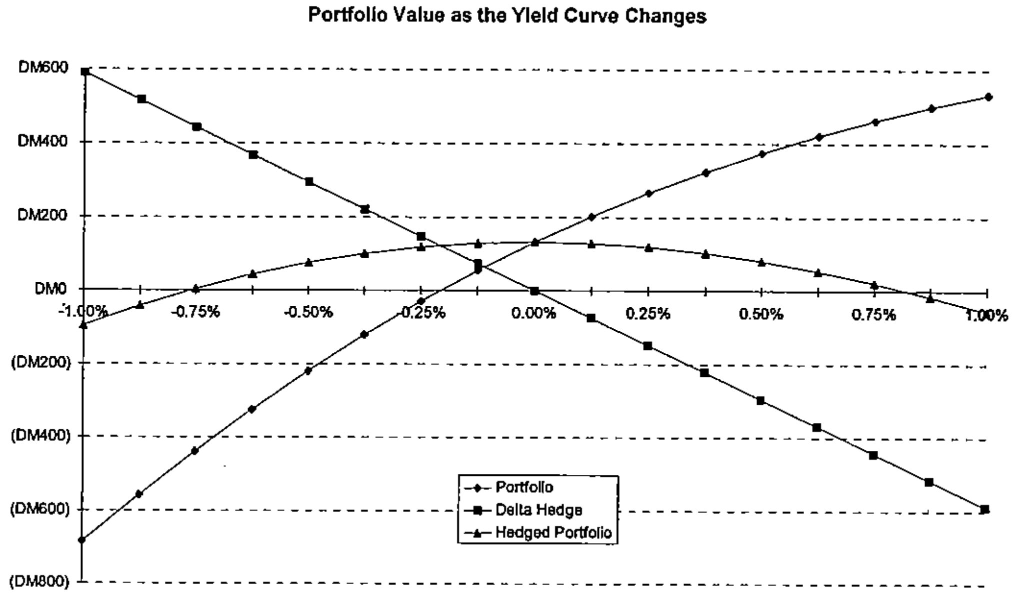


After delta-hedging our portfolio, we are left with a position which will lose money if rates move either up or down.

This is the problem of gamma. The option portfolio has gamma, as can be seen in the graph of its value at a range of possible interest rate change levels. As rates fall, the portfolio loses value at a faster and faster pace.

The futures and swap positions used to create the delta hedge have very little gamma. The delta hedge serves to hedge part of the risk for small movements in interest rates. But the delta hedge falls further and further behind if rates continue to fall.

The effectiveness of the delta hedge for small changes in interest rates can be seen by looking at the middle of the graph above:



For small movements of less than 0.75% in either direction, the delta hedge preserves part of the profit in the option portfolio.

But as rates continue to move, the delta hedge has to be rebalanced or the losses will grow.

The gamma is greatest on the 1×2 payer swaption we sold, and it makes a good example of the gamma risk. To delta hedge it, we need to pay fixed on par swaps.

As rates move down, the owner is less and less likely to exercise, and we need to unwind the hedge, in effect receiving fixed at lower and lower rates.

As rates move up, the owner is more and more likely to exercise, and we need to have more and more hedge, paying fixed at higher and higher rates.

Whichever way rates move, they are moving against us.

Vega

We can map the vega of the portfolio to the underlying cap and swaption volatilities used to price the caps, floors and swaptions. In effect we are creating a "vega map" in terms of the instruments which might be used to hedge it.

Instrument		Cap #1	Cap #2	Floor #1	Floor #2	Pay Swaption	Rec Swaption	TOTAL
Face Value		(DM15,000)	DM25,000	DM20,000	(DM50,000)	(DM25,000)	(DM50,000)	
Value		(DM141.60)	DM313.74	DM226.22	(DM325.05)	(DM180.57)	(DM760.07)	(DM867.33)
Cap Volatility								
1 Yr	19.50%	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00
2 Yr	17.80%	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00	DM0.00
3 Yr	17.20%	(DM9.80)	DM0.00	DM0.00	(DM28.97)	DM0.00	DM0.00	(DM38.76)
4 Yr	16.20%	DM0.00	DM0.00	DM19.182	DM0.00	DM0.00	DM0.00	DM19.18
5 Yr	15.50%	DM0.00	DM34.39	DM0.00	DM0.00	DM0.00	DM0.00	DM34.39
Swaption Volatility								
1x2	17.03%					(DM11.16)	DM0.00	(DM11.16)
1x4	15.64%					DM0.00	DM0.00	DM0.00
2x3	14.98%					DM0.00	(DM44.47)	(DM44.47)

In the map above, the vegas are mapped to the term volatilities of the various caps and floors, and swaptions. This is done by varying each volatility input by 1% and recalculating the option prices. Done in this manner, each instrument is sensitive to only one market volatility input.

If we assume that the term structure of volatility shifts up and down in parallel fashion, we can try and hedge the entire vega risk with a single purchased option. We can also hedge the vega of the portfolio to each cap and swaption volatility by using ATM caps, floors and swaption to hedge each bucket separately.

There are other ways to map vega, too. One common approach is to map to the implied forward forward volatilities, which would map part of each cap and floor to various forward forward volatilities.

It is also feasible to tie the swaptions to the map of forward forward volatilities, or even to map the whole portfolio to ATM swaption volatility, to use purchased swaptions to hedge short cap and floor vega.

Theta

The theta of the portfolio is DM0.88. This means that it will gainDM0.88 per day, assuming all else is constant. This will of course gather speed as the various instruments move nearer to expiry.

ADDING A SWAPTION

Now let us imagine we have a client who wishes to purchase DM100 mio of 1×4 payer swaption struck at 7.75%. The 1×4 forward swap rate is currently 7.6482% and the appropriate volatility is 15.64%.

According to the option pricing model, the value of this swaption is DM1,292.19. We agree to charge the client this price and he buys the swaption. What does this do to the portfolio we already had in place?

New Position and Delta Hedge

The portfolio now consists of the following:

Instrument	Position	Strike	Maturity	Face Value	Value
Cap #1	Short	7.50%	3	(DM15,000)	(DM141.60)
Cap #2	Long	8.50%	5	DM25,000	DM313.74
Floor #1	Long	6.50%	4	DM20,000	DM226.22
Floor #2	Short	6.25%	3	(DM50,000)	(DM325.05)
Pay Swaption	Short	7.50%	1×2	(DM25,000)	(DM180.57)
Rec. Swaption	Short	8.00%	2×3	(DM50,000)	(DM760.07)
Pay Swaption	Short	7.75%	1×4	(DM100,000)	(DM1,292.19)
Cash				DM2,523	DM2,292.19
TOTAL					**DM132.67**

An option pricing model tells you that the positions have the following sensitivities:

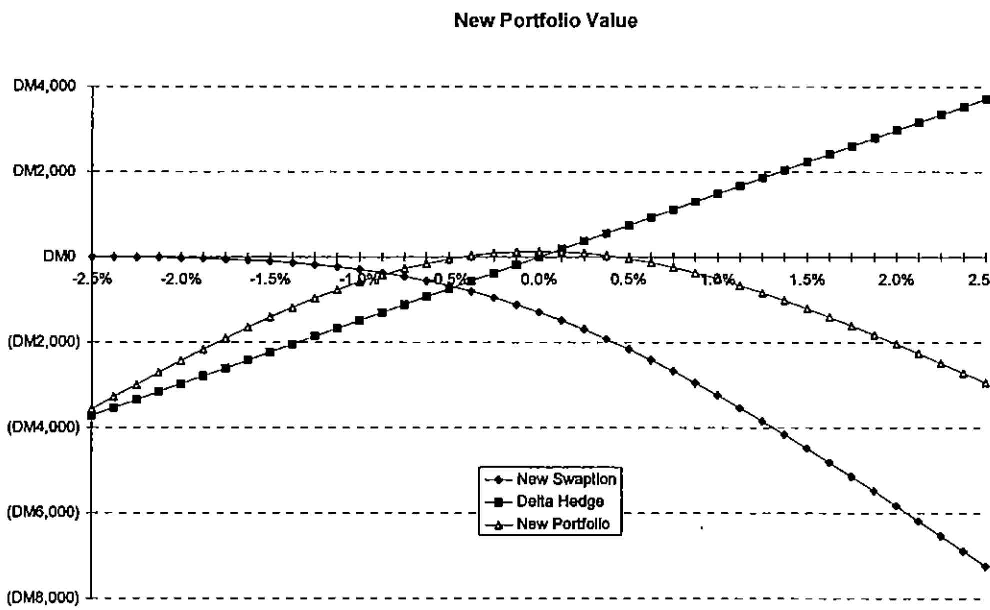
Instrument	Value	Δ	Γ	Vega	θ
Cap #1	(DM141.60)	(DM1.2197)	(DM0.0036)	(DM9.75)	DM0.14
Cap #2	DM313.74	DM2.5850	DM0.0074	DM34.07	(DM0.27)
Floor #1	DM226.22	(DM2.1518)	DM0.0075	DM19.02	(DM0.26)
Floor #2	(DM325.05)	DM3.6132	(DM0.0153)	(DM28.67)	DM0.55
Pay Swaption	(DM180.57)	(DM1.9387)	(DM0.0059)	(DM11.16)	DM0.26
Rec. Swaption	(DM760.07)	DM5.0098	(DM0.0121)	(DM44.47)	DM0.46
Pay Swaption	(DM1,292.19)	(DM14.8823)	(DM0.0498)	(DM91.49)	DM1.95
Cash	DM1,000.00	DM0.0000	DM0.0000	DM0.00	DM0.00
TOTAL	**DM132.67**	**(DM8.9845)**	**(DM0.0717)**	**(DM132.46)**	**DM2.83**

The delta has shifted from positive to negative, which means the overall position is now equivalent to paying fixed on a lot of swaps. The gamma has jumped, and will be working even more against us if rates move much at all.

The delta map now looks like this:

Input Rates		New Swaption	Delta Hedge	New Portfolio	New Hedge
Mar-95	94.83	DM1.34	53.4	DM1.5100	60.4
Jun-95	94.49	DM1.24	49.6	DM1.6382	65.5
Sep-95	94.10	DM1.24	49.6	DM1.8061	72.2
Dec-95	93.70	DM0.66	26.3	DM0.9701	38.8
Mar-96	93.36	DM0.00	0.0	DM0.0267	1.1
Jun-96	93.08	DM0.00	0.0	DM0.0147	0.6
2 Yr	6.4450%	DM0.03	DM0.158	(DM4.0755)	(DM22.299)
3 Yr	6.8450%	DM0.04	DM0.167	(DM2.6697)	(DM10.087)
4 Yr	7.0650%	DM0.06	DM0.178	(DM1.9053)	(DM5.594)
5 Yr	7.2150%	(DM19.28)	(DM46.920)	(DM5.6865)	(DM13.838)

Assuming we adjust our delta hedge to the amounts in the far right column, our portfolio now faces interest rate exposure even greater than before:



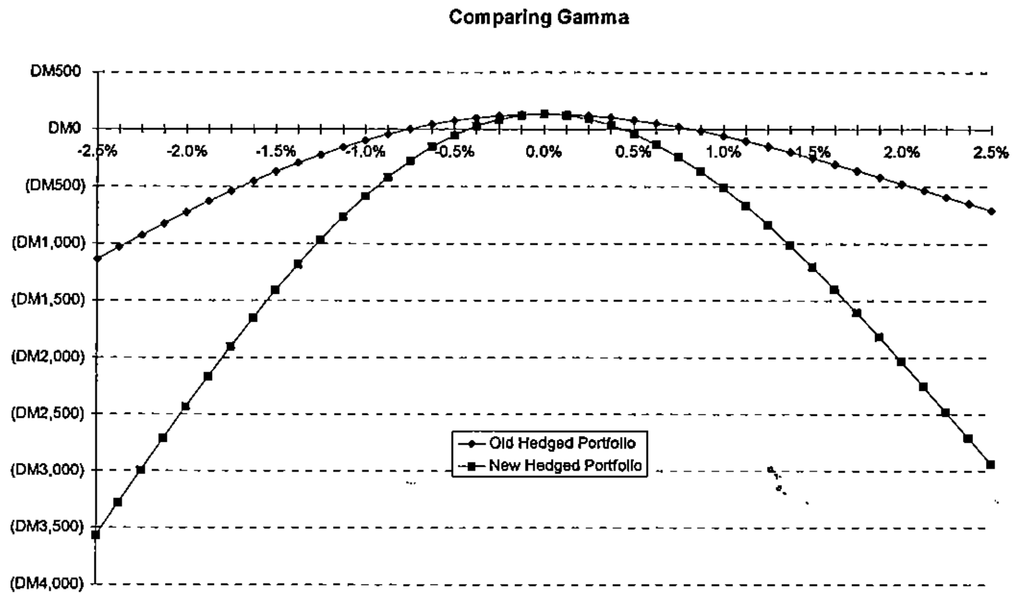
The new swaption will cost us money if rates rise, as its owner will be more and more likely to exercise into more and more profit (for him).

The delta hedge of the new swaption is paying fixed on DM47 mio of 5-year swap offset by buying large amounts of the four nearby futures. The combined effect is to leave us with a large amount of gamma.

Gamma Effect

The change in the gamma of the portfolio is from DM0.02 per basis point of rate movement to DM0.07 per basis point of rate movement.

Since we sold more options, gamma works against us harder than ever. The change is easy to see when we compare the delta-hedged portfolio value across a range of interest rate changes before and after the new swaption:



For option books consisting mostly of sold options, gamma is highly problematic.

There is only one way to hedge it: buy options sensitive to the same interest rate changes.

Hedging Vega

In order to hedge this option book more effectively, we need to buy options. Another approach used by many market participants is to begin with the vega, offsetting that and then hedging the residual delta risk.

This approach has one major positive: it also usually offsets the gamma risk.

It has one big negative, too: it means buying large quantities of options from the market. If client business is strong on both sides, i.e. clients are both buying and selling options to the bank, bid-offer spreads can mean nice profits with far less risk. If clients are mostly buying options from the bank, and the bank has to buy its hedges from the market, the profit of the client business will be far smaller.

In this case, let us imagine we wish to buy the same 1×4 payer swaptions to hedge the vega. How much do we need to buy?

The vega of the 1×4 payer swaption was DM91.49 per DM100,000. As a percent of face value, vega is thus: $\frac{91.49}{100{,}000} = 0.09149\%$.

The vega in the new portfolio is (DM132.46). To offset this much vega, we need to buy DM144,783 worth of the 1×4 payer swaption:

$$\text{Hedge} = \frac{132.46}{0.09149\%} = 144{,}783$$

At a price of 1.29219%, this will cost DM1,871. The portfolio now consists of:

Instrument	Position	Strike	Maturity	Face Value	Value
Cap #1	Short	7.50%	3	(DM15,000)	(DM141.60)
Cap #2	Long	8.50%	5	DM25,000	DM313.74
Floor #1	Long	6.50%	4	DM20,000	DM226.22
Floor #2	Short	6.25%	3	(DM50,000)	(DM325.05)
Pay Swaption	Short	7.50%	1×2	(DM25,000)	(DM180.57)
Rec. Swaption	Short	8.00%	2×3	(DM50,000)	(DM760.07)
Pay Swaption	Short	7.75%	1×4	DM44,783	DM578.68
Cash				DM421	DM421.32
TOTAL					**DM132.67**

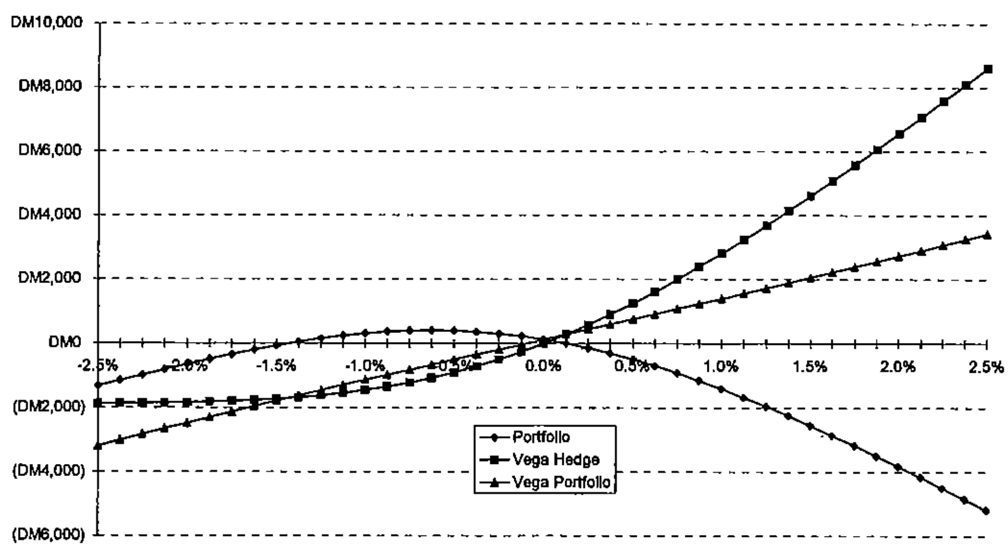
An option pricing model tells you that the positions have the following sensitivities:

Instrument	Value	Δ	Γ	Vega	θ
Cap #1	(DM141.60)	(DM1.2197)	(DM0.0036)	(DM9.75)	DM0.14
Cap #2	DM313.74	DM2.5850	DM0.0074	DM34.07	(DM0.27)
Floor #1	DM226.22	(DM2.1518)	DM0.0075	DM19.02	(DM0.26)
Floor #2	(DM325.05)	DM3.6132	(DM0.0153)	(DM28.67)	DM0.55
Pay Swaption	(DM180.57)	(DM1.9387)	(DM0.0059)	(DM11.16)	DM0.26
Rec. Swaption	(DM760.07)	DM5.0098	(DM0.0121)	(DM44.47)	DM0.46
Pay Swaption	DM578.68	DM6.6647	DM0.0223	DM40.97	(DM0.87)
Cash	DM421.32	DM0.0000	DM0.0000	DM0.00	DM0.00
TOTAL	**DM132.67**	**DM12.5626**	**DM0.0004**	**DM0.00**	**DM0.01**

This is a very interesting result. We have a lot of delta risk, but vega is down to zero, and the gamma is nearly all offset, as is theta.

This is shown on the graph following:

Hedging Vega by Buying Swaptions



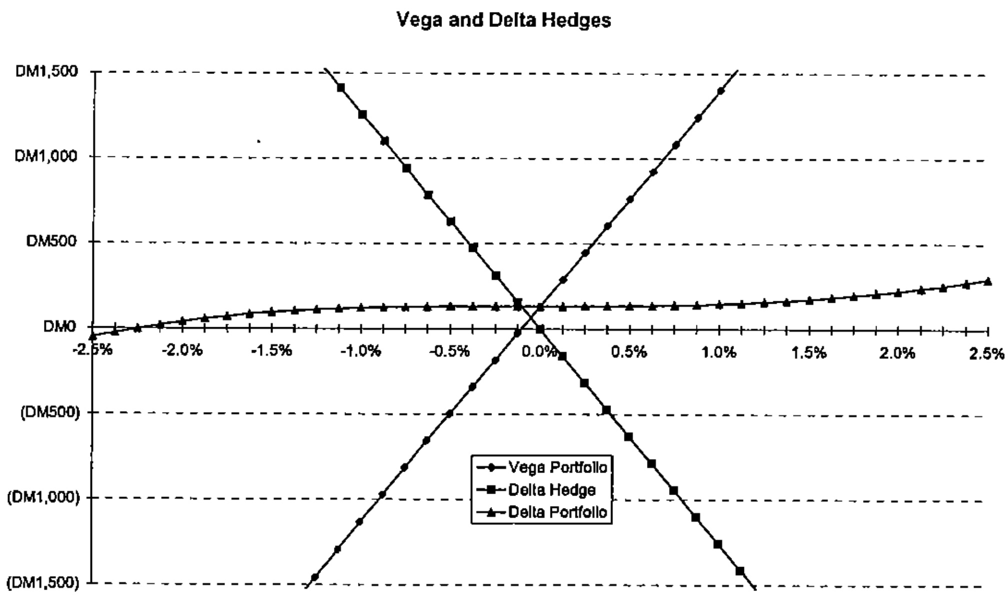
All that remains is to hedge the delta and see where we stand for various interest rate movements.

Hedging Delta

To hedge the delta, we need to map the delta to the market input instruments once more.

Input Rates		Previous Portfolio	New Vega Swaption	New Portfolio	New Delta Hedge
Mar-95	94.83	DM0.1738	(DM1.9346)	(DM1.7608)	-70.4
Jun-95	94.49	DM0.3972	(DM1.7967)	(DM1.3994)	-56.0
Sep-95	94.10	DM0.5664	(DM1.7949)	(DM1.2285)	-49.1
Dec-95	93.70	DM0.3128	(DM0.9516)	(DM0.6388)	-25.6
Mar-96	93.36	DM0.0267	DM0.0000	DM0.0267	1.1
Jun-96	93.08	DM0.0147	DM0.0000	DM0.0147	0.6
2 Yr	6.4450%	(DM4.1043)	(DM0.0418)	(DM4.1461)	(DM22.685)
3 Yr	6.8450%	(DM2.7138)	(DM0.0638)	(DM2.7776)	(DM10.495)
4 Yr	7.0650%	(DM1.9659)	(DM0.0878)	(DM2.0537)	(DM6.030)
5 Yr	7.2150%	DM13.5947	DM27.9159	DM41.5105	DM101.014

Assuming that is done and we adjust the delta hedge once more, we can measure the overall position across a range of interest rates:



Gamma

If we are willing to invest in the large payer swaption, gamma is hedged while we are hedging volatility.

Hedging delta only is better than not hedging at all. The same is true for hedging vega.

Hedging delta alone often increases gamma and vega.

Hedging vega and delta together may be a far more efficient way to protect the book's profits.

A well-constructed hedge of vega and gamma together is normally far more stable than hedging either one separately as both volatility and interest rate move.

No hedge is permanent. New deals are being booked all the time. Liquidity of futures, swaps, OTC options and futures options (where available), combined with a diversity of client deals leads to greater profitability from the option business.

Exhibit I

LIBOR-in-Arrears Swaps

For information contact:

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LIBOR-in-Arrears Swaps

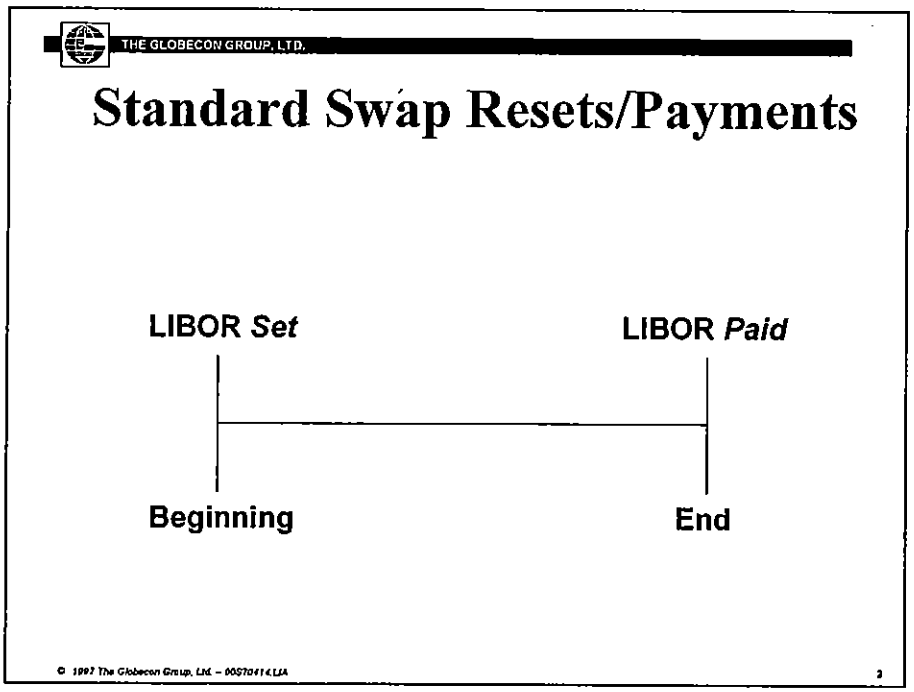
OVERVIEW

This module will cover LIBOR-in-Arrears swaps (LIA swaps), focusing on the application, pricing, and hedging of these swap structures. LIA swaps typically become interesting to clients whenever there is a steepness in the yield curve, either positive or negative. They are thus somewhat cyclical in their attractiveness, but are a fairly standard "product" for derivatives marketers.

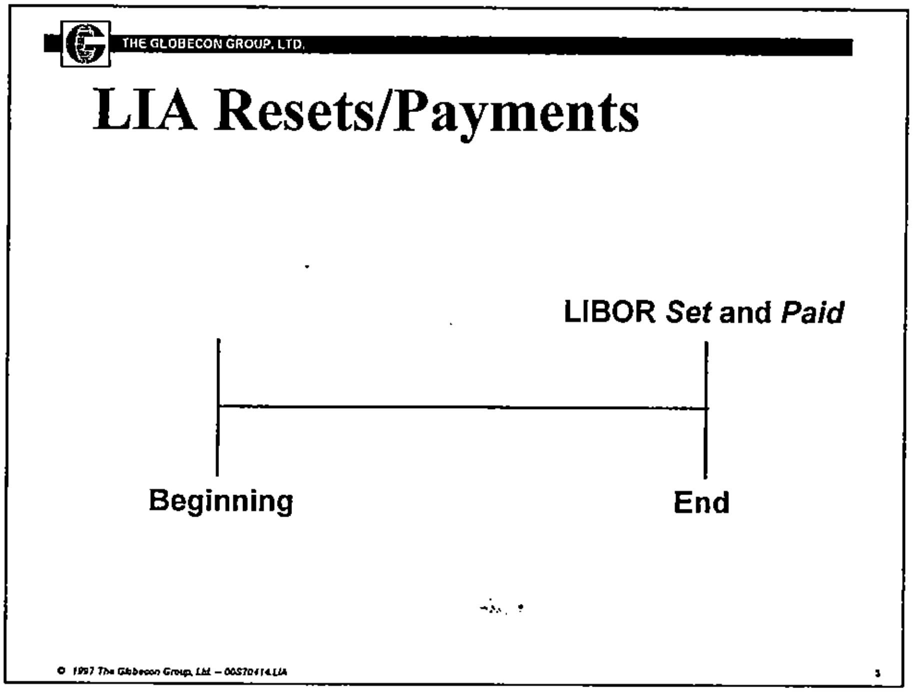
TAKING VIEWS ON YIELD CURVE STEEPNESS

Steep yield curve environments — both positive and negative — encourage paying or receiving LIBOR set at the end of the period instead of at the beginning.

The normal process of making payments in LIBOR **fixes** the rate at the beginning of the period and **pays** the rate at the end:



LIBOR-in-arrears payments change this pattern. LIBOR is **fixed at the end** of the period and **paid** at the end, two days later:



Normally, LIBOR is set using the rate for the same period beginning at the end of the relevant reset period.

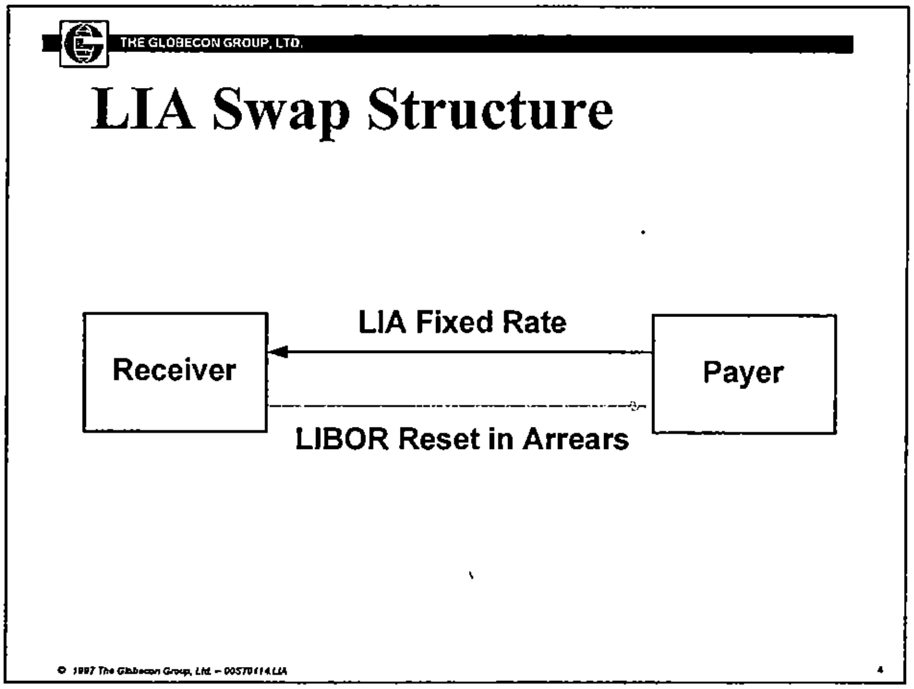
For example, if using six-month LIBOR resets, then the reset rate for the period just ending will be the market level of six-month LIBOR for the period just beginning.

This is not the only possible approach, however. One could choose to use the market rate for a longer or shorter LIBOR period, for example, or the five-year swap rate. This would produce a structure known as a "yield curve swap."

LIBOR-IN-ARREARS SWAP STRUCTURE

Basic Swap Structure

The basic LIA swap is very much like a standard interest rate swap:



The LIA fixed rate is about equal to the rate on a forward starting swap of the same tenor beginning one LIBOR reset period forward.

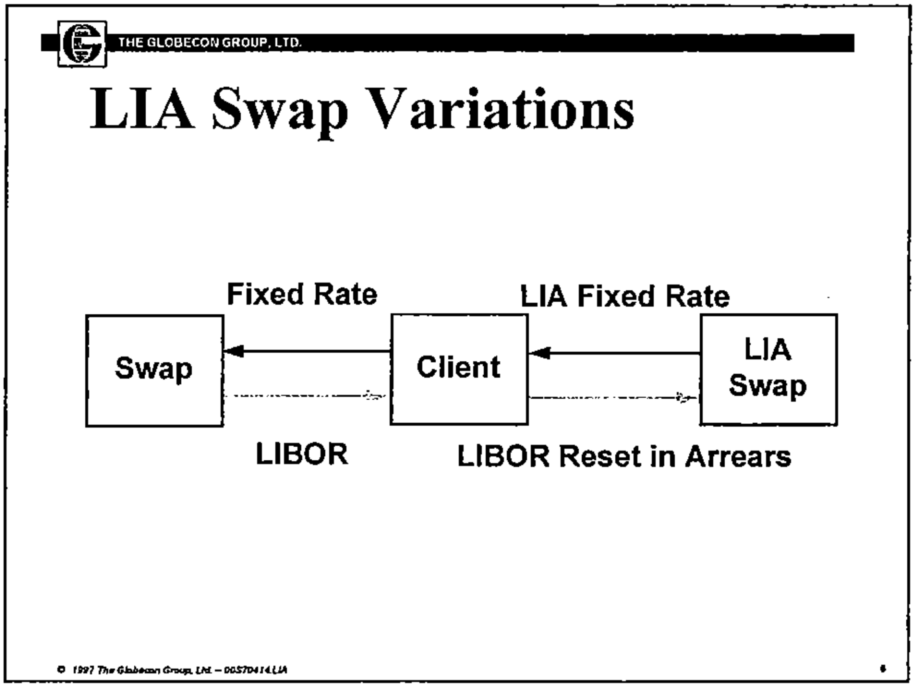
For an **upward-sloping** yield curve, the LIA fixed rate will be **higher** than the normal swap rate.

For a **downward-sloping** yield curve, the LIA fixed rate will be **lower** than the normal swap rate.

LIA SWAP VARIATIONS

Upward-Sloping Yield Curve

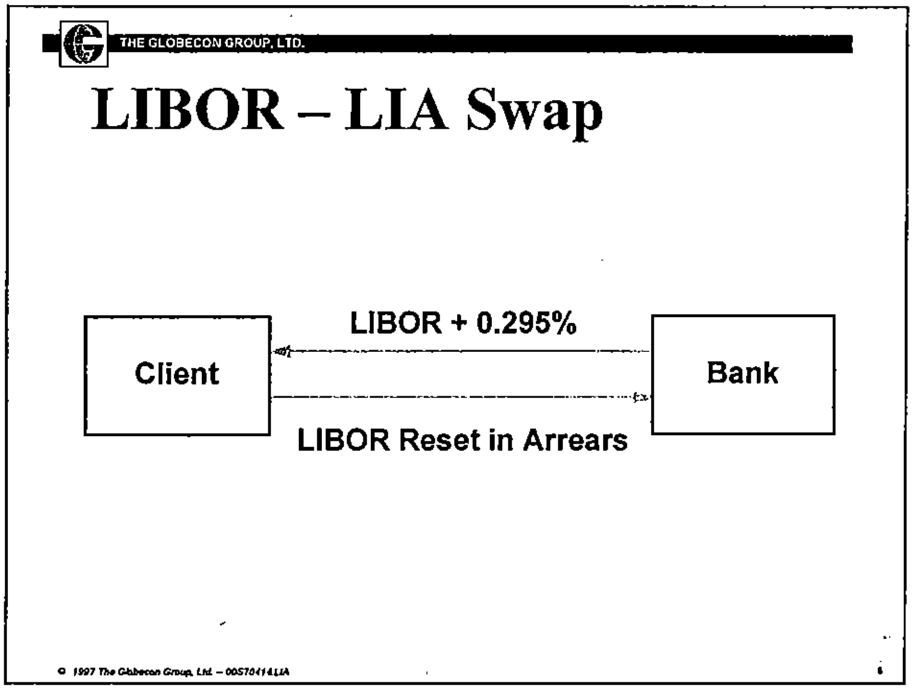
The LIA swap can be combined with a regular interest rate swap to exchange LIBOR in arrears against LIBOR set in the usual manner:



For an upward-sloping yield curve, the LIA fixed rate will be higher than the normal swap rate by a fixed amount.

For example, if the LIA swap rate is 5.95% and the normal swap rate is 5.655%, the client could receive the LIA spread of 0.295% each period.

This would produce the following position for the client:

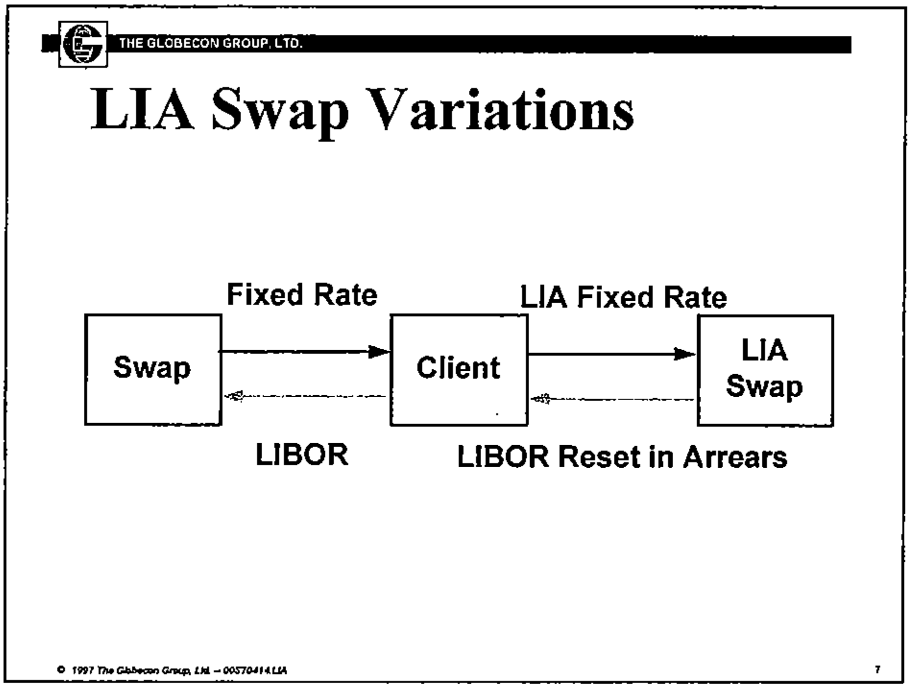


As long as LIBOR does not rise by more than 0.295% on average over each reset period, the client would come out ahead.

By taking this position, the client is stating the view that the forward curve "forecasts" LIBOR rising faster and sooner than the client believes will happen. The curve is too steeply upward sloping.

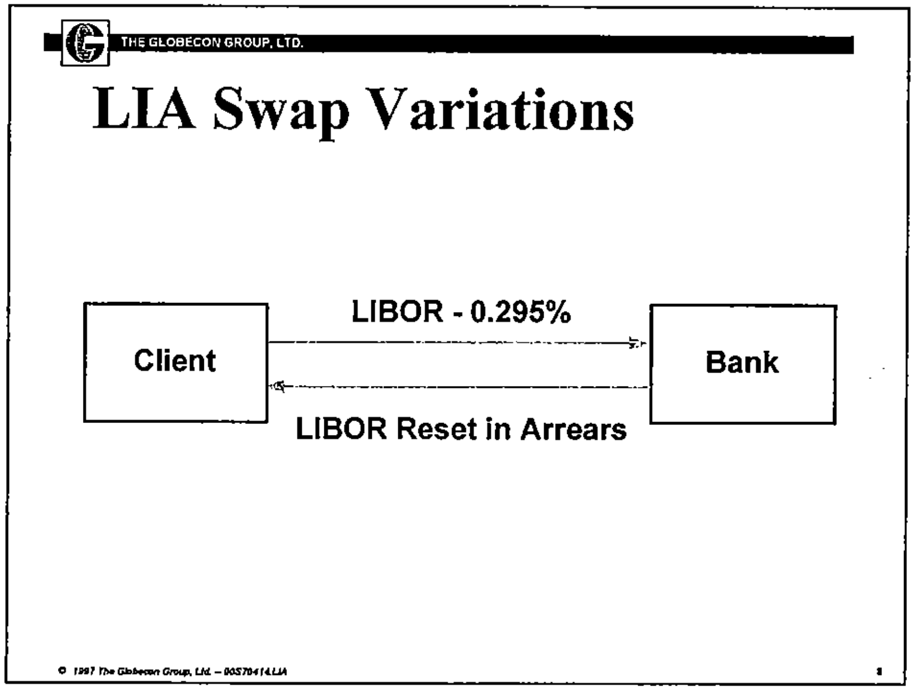
Downward-Sloping Yield Curve
For a downward-sloping yield curve, the LIA swap rate would be lower than the normal swap rate.

The client might prefer the following position:



If the LIA swap rate was 5.36% and the normal swap rate was 5.655%, the client would pay the LIA spread of –0.295% each period.

This would produce the following position for the client:



As long as LIBOR did not fall by more than 0.295% on average over each reset period, the client would come out ahead.

By taking this position, the client is stating the view that the forward curve "forecasts" LIBOR falling faster and sooner than the client believes will happen. The curve is too steeply downward sloping.

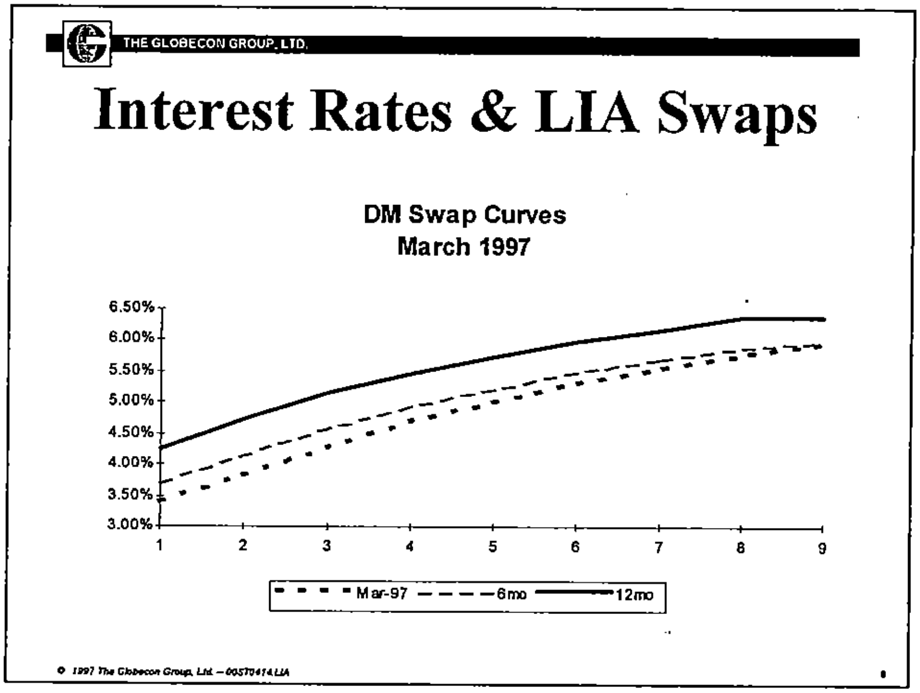
INTEREST RATES VIEWS AND LIA SWAPS

Following is an example of how to price LIA swaps. In March 1997 the following Deutschemark swap curves were available from the market:

Date	Days	Swaps	PV Factors	DM FRAs	Forward Swaps	
					6 Months	12 months
3/17/97			1.000000			
9/17/97	184		0.983564	3.3422%		
3/17/98	181	3.3867%	0.966803	3.4674%		
9/17/98	184		0.947657	4.0405%	3.7139%	
3/17/99	181	3.8350%	0.927359	4.3777%		4.2533%
9/17/99	184		0.904925	4.9582%	4.1516%	
3/17/00	182	4.2750%	0.881347	5.3505%		4.7247%
9/17/00	184		0.856352	5.8375%	4.5834%	
3/17/01	181	4.6950%	0.830689	6.1788%		5.1570%
9/17/01	184		0.805421	6.2744%	4.9399%	
3/17/02	181	5.0150%	0.780031	6.5101%		5.4621%
9/17/02	184		0.754275	6.8292%	5.2275%	
3/17/03	181	5.3050%	0.728655	7.0321%		5.7411%
9/17/03	184		0.703044	7.2858%	5.4844%	
3/17/04	182	5.5650%	0.677646	7.4961%		5.9920%
9/17/04	184		0.653725	7.3182%	5.6942%	
3/17/05	181	5.7550%	0.630363	7.4125%		6.1663%
9/17/05	184		0.605947	8.0585%	5.8792%	
3/17/06	181	5.9650%	0.582149	8.1759%		6.3703%
9/17/06	184		0.564307	6.3237%	5.9851%	
3/17/07	181	5.9950%	0.547241	6.2372%		6.3710%

- Swaps are quoted on a 30/360 basis.
- The three-year swap rate is 4.275%.
- The three-year swap rate six months forward is 4.583%.
- The three-year swap rate one year forward is 5.157%.

Graphing these rates illustrates them more clearly:



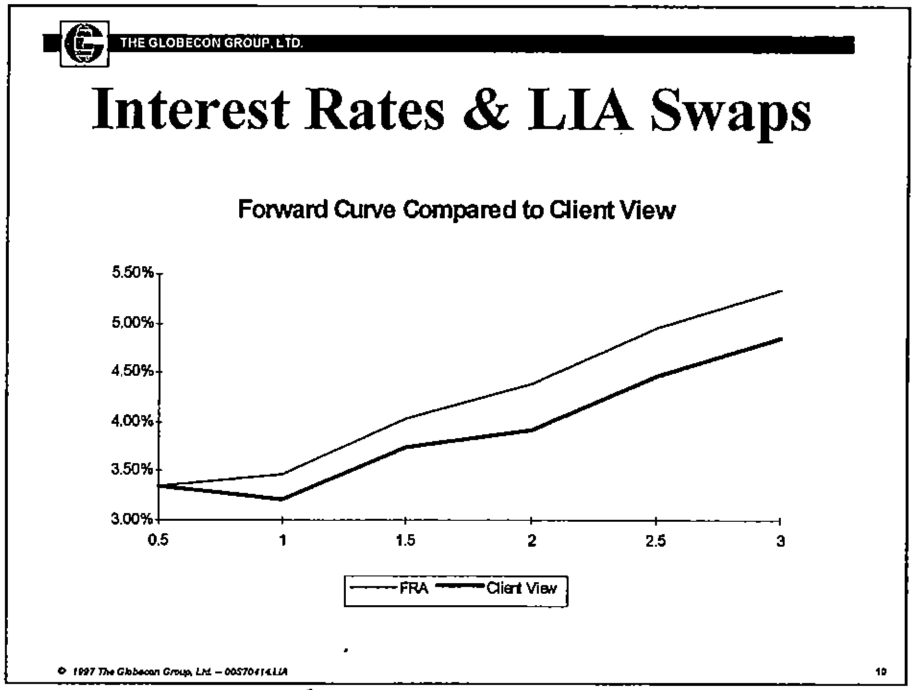
The swap curve remains upward sloping, with rates across the medium-term end of the curve rising steadily.

Remember that a LIA swap will have a fixed rate more or less equal to the forward swap rate one LIBOR reset period forward.

- The rate on a three-year LIA swap with six-month LIBOR resets would be about 4.58%.
- The rate using 12-month LIBOR resets would be about 5.157%.

The forward curve is very steeply upward sloping.

This suggests using one of the variations discussed above for clients which do not believe the short end of the curve will rise as quickly as the yield curve forecasts.

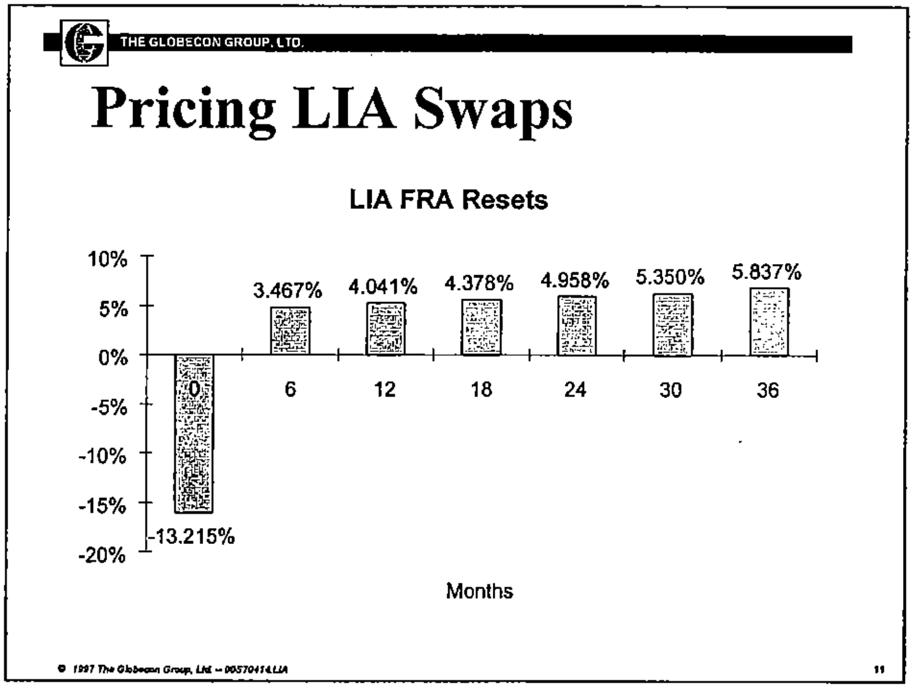


Let clients pay six-month LIBOR set in arrears and receive six-month LIBOR plus a spread of roughly 0.486% for three years.

PRICING LIA SWAPS

Pricing a LIA swap is rather like pricing a regular interest rate swap, except that the mark-to-market values for LIBOR are taken from the period **beginning** at each payment date, rather than ending at each payment date.

To price the three-year LIA swap, for example, use the strip of six-month FRAs beginning with the 6×12 FRA, but applying it to the 0×6 period:



The graph above uses the following rates:

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Pricing LIA Swaps

Date	*Days*	*Swaps*	*PV Factors*	*6-mo LIA FRA*	*FRA CF PV*
3/17/97			1.000000		13.2150%
9/17/97	184		0.983564	3.467%	1.7431%
3/17/98	181	3.387%	0.966803	4.041%	1.9641%
9/17/98	184		0.947657	4.378%	2.1204%
3/17/99	181	3.835%	0.927359	4.958%	2.3118%
9/17/99	184		0.904925	5.350%	2.4747%
3/17/00	182	4.275%	0.881347	5.837%	2.6010%

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The present value of the FRA strip is calculated in the usual way, using actual days over 360 and the respective PV factors, according to the following formula, where *s* refers to the strip of semiannual periods. The only difference is that the formula uses the number of days from the period just ending instead of the number of days in the period just beginning:

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Pricing LIA Swaps

$$\text{FRA PV} = \sum_{s=1}^{n}\left(\text{FRA}_s \times \frac{\text{Days}_s}{360} \times \text{PVf}_s\right)$$

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For example, the PV for the first six-month period is 1.743%:

$$1.743\% = 3.467\% \times \frac{184}{360} \times 0.9836$$

In this case, on the fixed-rate side of the swap use the actual number of workdays on a 30/360 basis. The day count for the annual fixed-rate payments has to take into account the number of 30/360 days in each annual period.

Now solve for a single fixed-rate payment which, when adjusted for the number of 30/360 days from year to year, returns the same PV as the strip of FRAs above.

$$PV = \sum_{t=1}^{n} \left(Pmnt \times \frac{30/360\ Days_t}{360} \times PVf_t \right)$$

Since Pmnt is the same every period, it is possible to solve for it directly:

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Pricing LIA Swaps

$$Pmnt = \frac{PV}{\sum_{t=1}^{n} \left(PVf_t \times \frac{30/360\ Days_t}{360} \right)}$$

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In other words, the LIA swap fixed-rate payment is equal to the PV of the floating-rate cash flows divided by the sum of the relevant annual PV factors at 12 months, 24 months, and 36 months.

Calculate the rate:

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$$\text{Pmnt} = \frac{13.215\%}{(0.9668 \times \frac{360}{360} + 0.9273 \times \frac{360}{360} + 0.8813 \times \frac{360}{360})}$$

Pmnt = 4.761%

 15

This rate is close to the rate on a three-year swap beginning in six months, 4.583%.

It is some 0.486% higher than the rate on the normal three-year, which is 4.275%. This is the source of the LIA swap spread mentioned above.

HEDGING THE LIA SWAP

Hedging this swap using FRAs or other swaps requires consideration of LIBOR volatility.

This is because the FRAs are being used one period early.

The FRA values can be hedged, but the discount rates that will be used to move the FRA cash flows from the end of the period to the beginning cannot.

Thus, the future value of the LIBOR cash flows can be locked in, but not their "settlement values."

This will be based on the actual level of LIBOR, which is, of course, unknown.

But a range can be estimated using the FRAs and market volatility.

In this case, assuming a level of volatility for each FRA of 20% (probably a little high), adjust the expected future LIBOR levels as shown on the following page:

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Hedging LIA Swaps

Period	*LIA FRA*	*Lower Limit*	*Upper Limit*
0×6	3.467%	2.975%	3.960%
6×12	4.041%	3.471%	4.610%
12×18	4.378%	3.756%	4.999%
18×24	4.958%	4.260%	5.657%
24×30	5.350%	4.591%	6.110%
30×36	5.837%	5.013%	6.662%

 14

In each case, the lower and upper limits are calculated using the following

relationships: $\sigma = \frac{1}{2} \times \frac{1}{\sqrt{\text{time}}} \times \ln\left(\frac{\text{Upper Limit}}{\text{Lower Limit}}\right)$

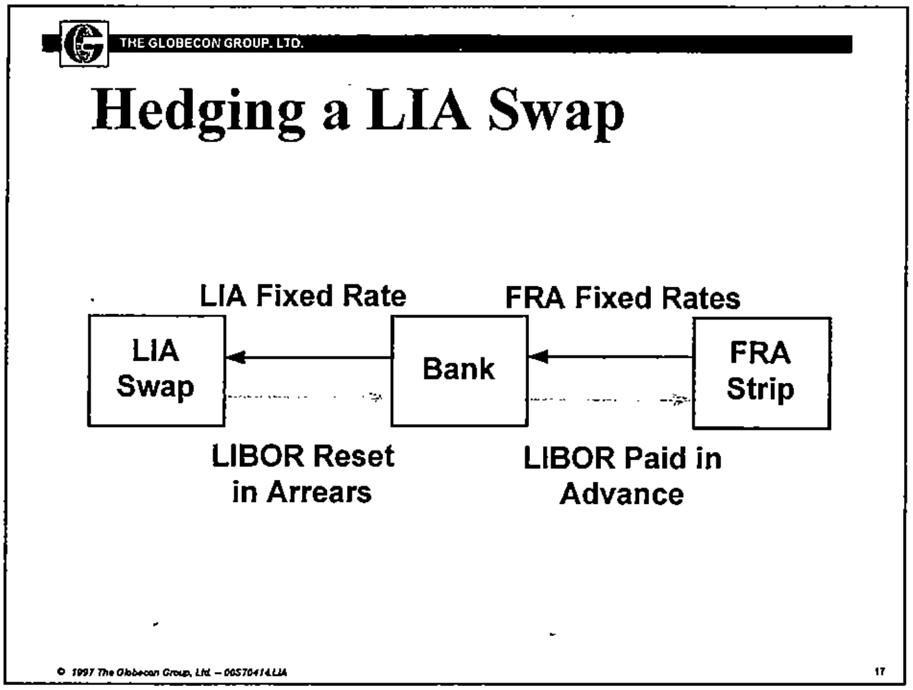
$$\text{FRA} = \text{Lower Limit} \times 50\% + \text{Upper Limit} \times 50\%$$

Combining these two equations leads to the following:

$$\text{Lower Limit} = \frac{2 \times \text{FRA}}{1 + e^{\left(2 \times \sigma \times \sqrt{t}\right)}}$$

$$\text{Upper Limit} = \text{Lower Limit} \times e^{\left(2 \times \sigma \times \sqrt{t}\right)}$$

Imagine the bank has taken the following position in a LIA swap:



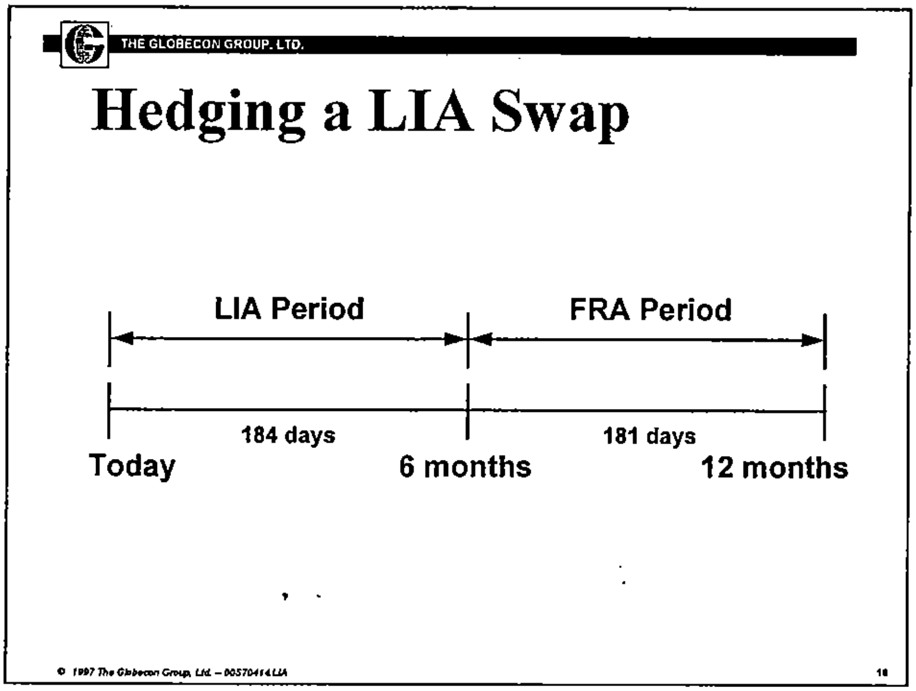
The strip of FRAs is a way to lock in the value of the LIBOR payments the bank will receive.

Selling the FRA for the LIA period beginning at the payment date provides a hedge of the rate used for the LIA payment.

The bank will receive a payment from the LIA swap based on the rate of LIBOR for the following six months. This is the same rate the FRA will refer to.

The LIA swap payment will take place at the beginning of the period. The bank will settle the FRA at the beginning of each period, too, so it will have about the cash flow it needs, but the FRA settlement will be for the PV of the cash flow at the beginning of the period.

These relationships can be depicted on the following time line for the very first six-month period in the deal, the 0×6 period:



The floating-rate payment received from the LIA swap will use the market level for six-month LIBOR applied to the first six-month period with 184 days:

$$\text{LIA Payment} = \text{LIBOR} \times \frac{184}{360} \times \text{Swap Notional Principal}$$

LIA Swaps and "Quanto Risk"

The FRA net payment will also use the market level of six-month LIBOR in six months. But it will apply it to the 6×12 period of 181 days and discount it to the beginning of the period at six months as follows:

$$\text{FRA Payment} = \frac{\text{LIBOR} \times \frac{181}{360}}{\left(1 + \text{LIBOR} \times \frac{181}{360}\right)} \times \text{FRA Notional Principal}$$

To make these two payments more or less equal, raise the notional principal on the FRA by a factor equal to the discount and adjust for the different number of actual days:

$$\text{FRA Payment} = \text{LIA Payment}$$

$$NP_{FRA} \times \frac{L \times \frac{181}{360}}{\left(1 + L \times \frac{181}{360}\right)} = NP_{Swap} \times L \times \frac{184}{360}$$

$$NP_{FRA} = NP_{Swap} \times \frac{184}{181} \times \left(1 + L \times \frac{181}{360}\right)$$

This adjustment will be different for each period, and will change as LIBOR changes. This makes it difficult to calculate the amount of adjustment needed in advance.

While it is possible to fix the FRA rate against LIBOR each period, it is not possible to fix the market rate at which the payment will be discounted to the beginning of each period. This adds a second degree of uncertainty into the hedge.

This uncertainty is a form of ***quanto risk***, or risk that cannot be hedged in advance.

It is also reasonable to assume that the discount rate will fluctuate from the lower to the upper limits calculated above, and calculate an adjustment to the LIA swap rate that keeps the bank whole when the FRA discount rates change to the extremes of one standard deviation.

In this case the adjusted rate is less than two basis points different. The effect is minimal.

This is well known among FRA traders using futures contracts to hedge their FRA positions.

It results from the fact that FRAs have convexity while futures contracts are linear.

FRA convexity results from the fact that the payout on the FRA is a function of changing LIBOR rates, which result in a changing discount rate.

Futures variation margin is not discounted from the end of the period to the beginning, but paid daily over the life of the contract.

Futures would thus represent a more interesting hedge, if there existed liquid futures contracts covering the full term of the LIA swap.

LIBOR-in-Arrears Swaps

This module covers LIBOR-in-arrears swaps: applications, pricing, and hedging. It can be used in any slightly more advanced FRM or fixed income workshop. A working knowledge of PV factors and basic swap pricing is necessary.

This module is a natural follow-up to basic swap pricing and logic, and can be used as part of an overall discussion of "advanced" swaps or variations of interest rate swaps.

The module should take approximately 30 minutes to cover.

For information contact:

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(212) 227-0443 fax

Exhibit M

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Serials Database (Author Search)
Search For: GLOBECON GROUP, LTD
24 Items
For a list of commonly used abbreviations that appear in the catalog record, click here.

Title:	Skills & applications
Additional Information:	: the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note:	Monthly. Running ti.: S & A.
Holdings:	* acGlobecon Group, Ltd.
	TX 5-271-923. 23Jun98; Apr98. PUB 1Apr98;
	TX 5-271-923. 23Jun98; May98. PUB 1May98;
	TX 5-271-923. 23Jun98; Jun98. PUB 1Jun98;
	TX 5-318-753. 19Mar01; Jan99. PUB 1Jan99;
	TX 5-318-753. 19Mar01; Feb99. PUB 1Feb99;
	TX 5-318-753. 19Mar01; Mar99. PUB 1Mar99;
	TX 5-318-752. 19Mar01; Apr99. PUB 1Apr99;
	TX 5-318-752. 19Mar01; May99. PUB 1May99;
	TX 5-318-752. 19Mar01; Jun99. PUB 1Jun99;

Title:	Skills & applications
Additional Information:	: the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note:	Monthly. Running ti.: S & A.
Holdings:	* the acGlobecon Group, Ltd.
	TX 2-168-596. 19Oct87; Oct87. DCR 1987; PUB 7Oct87;

Title:	Skills & applications
Additional Information:	: the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note:	Monthly. Running ti.: S & A.
Holdings:	* acGlobecon Group, Ltd.
	TX 3-489-073. 16Nov92; Jan92. PUB 1Jan92;

TX 3-489-073. 16Nov92; Feb92. PUB 1Feb92;
TX 3-489-073. 16Nov92; Mar92. PUB 1Mar92;
TX 3-489-074. 16Nov92; Apr92. PUB 1Apr92;
TX 3-489-074. 16Nov92; May92. PUB 1May92;
TX 3-489-074. 16Nov92; Jun92. PUB 1Jun92;
TX 3-489-075. 16Nov92; Jul92. PUB 1Jul92;
TX 3-489-075. 16Nov92; Aug92. PUB 1Aug92;
TX 3-489-075. 16Nov92; Sep92. PUB 1Sep92;
TX 3-489-076. 16Dec92; Oct92. PUB 1Oct92;
TX 3-489-076. 16Dec92; Nov92. PUB 1Nov92;
TX 3-489-076. 16Dec92; Dec92. PUB 1Dec92;
TX 3-539-437. 5May93; Jan93. PUB 1Jan93;
TX 3-539-437. 5May93; Feb93. PUB 1Feb93;
TX 3-539-437. 5May93; Mar93. PUB 1Mar93;

Title: Skills & applications
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly. Running ti.: S & A.
Holdings: * acGlobecon Group, Ltd.
TX 3-629-344. 27Sep93; Jul93. PUB 1Jul93;
TX 3-629-344. 27Sep93; Aug93. PUB 1Aug93;
TX 3-629-344. 27Sep93; Sep93. PUB 1Sep93;

Title: Skills & applications
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly. Running ti.: S & A.
Holdings: * acGlobecon Group, Ltd.
TX 3-668-333. 19Jul93; Apr93. PUB 1Apr93;
TX 3-668-333. 19Jul93; May93. PUB 1May93;
TX 3-668-333. 19Jul93; Jun93. PUB 1Jun93;
TX 3-694-632. 29Dec93; Oct93. PUB 1Oct93;

TX 3-694-632. 29Dec93; Nov93. PUB 1Nov93;
TX 3-694-632. 29Dec93; Dec93. PUB 1Dec93;
TX 3-740-320. 21Mar94; Jan94. PUB 1Jan94;
TX 3-740-320. 21Mar94; Feb94. PUB 1Feb94;
TX 3-740-320. 21Mar94; Mar94. PUB 1Mar94;

Title: Skills & applications
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly. Running ti.: S & A.
Holdings: * acGlobecon Group, Ltd.
TX 3-837-210. 1Jul94; Apr94. PUB 1Apr94;
TX 3-837-210. 1Jul94; May94. PUB 1May94;
TX 3-837-210. 1Jul94; Jun94. PUB 1Jun94;

Title: Skills & applications
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly. Running ti.: S & A.
Holdings: * acGlobecon Group, Ltd.
TX 4-005-329. 26Apr95; Oct94. PUB 1Oct94;
TX 4-005-329. 26Apr95; Nov94. PUB 1Nov94;
TX 4-005-329. 26Apr95; Dec94. PUB 1Dec94;
TX 4-005-331. 26Apr95; Jan95. PUB 1Jan95;
TX 4-005-331. 26Apr95; Feb95. PUB 1Feb95;
TX 4-005-331. 26Apr95; Mar95. PUB 1Mar95;

Title: Skills & applications
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly. Running ti.: S & A.
Holdings: * acGlobecon Group, Ltd.
TX 4-156-513. 11Oct95; Jul94. PUB 1Jul94;
TX 4-156-513. 11Oct95; Aug94. PUB 1Aug94;

TX 4-156-513. 11Oct95; Sep94. PUB 1Sep94;
TX 4-165-250. 4Dec95; Apr95. PUB 1Apr95;
TX 4-165-250. 4Dec95; May95. PUB 1May95;
TX 4-165-250. 4Dec95; Jun95. PUB 1Jun95;
TX 4-165-285. 6Dec95; Jul95. PUB 1Jul95;
TX 4-165-285. 6Dec95; Aug95. PUB 1Aug95;
TX 4-165-285. 6Dec95; Sep95. PUB 1Sep95;
TX 4-184-331. 3Jan96; Oct95. PUB 1Oct95;
TX 4-184-331. 3Jan96; Nov95. PUB 1Nov95;
TX 4-184-331. 3Jan96; Dec95. PUB 1Dec95;

Title: Skills & applications
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly. Running ti.: S & A.
Holdings: * acGlobecon Group, Ltd.
TX 4-271-754. 6Jun96; Jan96. PUB 1Jan96;
TX 4-271-754. 6Jun96; Feb96. PUB 1Feb96;
TX 4-271-754. 6Jun96; Mar96. PUB 1Mar96;
TX 4-311-441. 12Aug96; Apr96. PUB 1Apr96;
TX 4-311-441. 12Aug96; May96. PUB 1May96;
TX 4-311-441. 12Aug96; Jun96. PUB 1Jun96;

Title: Skills & applications
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly. Running ti.: S & A.
Holdings: * acGlobecon Group, Ltd.
TX 4-673-999. 14Apr98; Oct97. PUB 1Oct97;
TX 4-673-999. 14Apr98; Nov97. PUB 1Nov97;
TX 4-673-999. 14Apr98; Dec97. PUB 1Dec97;
TX 4-650-859. 14Apr98; Jan98. PUB 1Jan98;
TX 4-650-859. 14Apr98; Feb98. PUB 1Feb98;

TX 4-650-859. 14Apr98; Mar98. PUB 1Mar98;

Title: Skills & applications
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly. Running ti.: S & A.
Holdings: * acGlobecon Group, Ltd.
TX 4-798-123. 8Dec97; Jan97. PUB 1Jan97;
TX 4-798-123. 8Dec97; Feb97. PUB 1Feb97;
TX 4-798-123. 8Dec97; Mar97. PUB 1Mar97;
TX 4-798-124. 8Dec97; Apr97. PUB 1Apr97;
TX 4-798-124. 8Dec97; May97. PUB 1May97;
TX 4-798-124. 8Dec97; Jun97. PUB 1Jun97;
TX 4-798-125. 8Dec97; Jul97. PUB 1Jul97;
TX 4-798-125. 8Dec97; Aug97. PUB 1Aug97;
TX 4-798-125. 8Dec97; Sep97. PUB 1Sep97;

Title: Skills & applications
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly. Running ti.: S & A.
Holdings: * acGlobecon Group, Ltd.
TX 4-911-467. 4Dec98; Jul98. PUB 1Jul98;
TX 4-911-467. 4Dec98; Aug98. PUB 1Aug98;
TX 4-911-467. 4Dec98; Sep98. PUB 1Sep98;
TX 4-911-465. 12Feb99; Oct98. PUB 1Oct98;
TX 4-911-465. 12Feb99; Nov98. PUB 1Nov98;
TX 4-911-465. 12Feb99; Dec98. PUB 1Dec98;

Title: Techniques & products
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly.
Holdings: * the acGlobecon Group, Ltd.

TX 2-173-454. 19Oct87; Oct87. DCR 1987; PUB 7Oct87;

Title: Techniques & products
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly.
Holdings: * acGlobecon Group, Ltd.
TX 3-474-283. 16Nov92; Jan92. PUB 1Jan92;
TX 3-474-283. 16Nov92; Feb92. PUB 1Feb92;
TX 3-474-283. 16Nov92; Mar92. PUB 1Mar92;
TX 3-489-077. 16Nov92; Apr92. PUB 1Apr92;
TX 3-489-077. 16Nov92; May92. PUB 1May92;
TX 3-489-077. 16Nov92; Jun92. PUB 1Jun92;
TX 3-469-993. 16Nov92; Jul92. PUB 1Jul92;
TX 3-469-993. 16Nov92; Aug92. PUB 1Aug92;
TX 3-469-993. 16Nov92; Sep92. PUB 1Sep92;
TX 3-474-282. 16Dec92; Oct92. PUB 1Oct92;
TX 3-474-282. 16Dec92; Nov92. PUB 1Nov92;
TX 3-474-282. 16Dec92; Dec92. PUB 1Dec92;
TX 3-539-436. 5May93; Jan93. PUB 1Jan93;
TX 3-539-436. 5May93; Feb93. PUB 1Feb93;
TX 3-539-436. 5May93; Mar93. PUB 1Mar93;

Title: Techniques & products
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly.
Holdings: * acGlobecon Group, Ltd.
TX 3-629-345. 27Sep93; Jul93. PUB 1Jul93;
TX 3-629-345. 27Sep93; Aug93. PUB 1Aug93;
TX 3-629-345. 27Sep93; Sep93. PUB 1Sep93;

Title: Techniques & products

Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly.
Holdings: * acGlobecon Group, Ltd.
TX 3-668-332. 19Jul93; Apr93. PUB 1Apr93;
TX 3-668-332. 19Jul93; May93. PUB 1May93;
TX 3-668-332. 19Jul93; Jun93. PUB 1Jun93;
TX 3-694-633. 29Dec93; Oct93. PUB 1Oct93;
TX 3-694-633. 29Dec93; Nov93. PUB 1Nov93;
TX 3-694-633. 29Dec93; Dec93. PUB 1Dec93;

Title: Techniques & products
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly.
Holdings: * acGlobecon Group, Ltd.
TX 3-837-610. 21Mar94; Jan94. PUB 1Jan94;
TX 3-837-610. 21Mar94; Feb94. PUB 1Feb94;
TX 3-837-610. 21Mar94; Mar94. PUB 1Mar94;
TX 3-818-910. 1Jul94; Apr94. PUB 1Apr94;
TX 3-818-910. 1Jul94; May94. PUB 1May94;
TX 3-818-910. 1Jul94; Jun94. PUB 1Jun94;
TX 3-916-045. 11Oct94; Jul94. PUB 1Jul94;
TX 3-916-045. 11Oct94; Aug94. PUB 1Aug94;
TX 3-916-045. 11Oct94; Sep94. PUB 1Sep94;

Title: Techniques & products
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly.
Holdings: * acGlobecon Group, Ltd.
TX 4-005-330. 26Apr95; Oct94. PUB 1Oct94;
TX 4-005-330. 26Apr95; Nov94. PUB 1Nov94;
TX 4-005-330. 26Apr95; Dec94. PUB 1Dec94;

Title: Techniques & products
Additional Information: : the finance update service of the Globecon Group, Ltd. -- Oct. 1987-.
Note: Monthly.
Holdings: * acGlobecon Group, Ltd.
TX 4-048-042. 26Apr95; Jan95. (CO corres.) PUB 1Jan95;
TX 4-048-042. 26Apr95; Feb95. (CO corres.) PUB 1Feb95;

Title: Techniques & solutions
Additional Information: .
Holdings: * acGlobecon Group, Ltd.
TX 4-165-251. 4Dec95; Apr95. PUB 1Apr95;
TX 4-165-251. 4Dec95; May95. PUB 1May95;
TX 4-165-251. 4Dec95; Jun95. PUB 1Jun95;
TX 4-165-261. 6Dec95; Jul95. PUB 1Jul95;
TX 4-165-261. 6Dec95; Aug95. PUB 1Aug95;
TX 4-165-261. 6Dec95; Sep95. PUB 1Sep95;
TX 4-184-332. 3Jan96; Oct95. PUB 1Oct95;
TX 4-184-332. 3Jan96; Nov95. PUB 1Nov95;
TX 4-184-332. 3Jan96; Dec95. PUB 1Dec95;

Title: Techniques & solutions
Additional Information: .
Holdings: * acGlobecon Group, Ltd.
TX 4-271-755. 6Jun96; Jan96. PUB 1Jan96;
TX 4-271-755. 6Jun96; Feb96. PUB 1Feb96;
TX 4-271-755. 6Jun96; Mar96. PUB 1Mar96;
TX 4-319-321. 12Aug96; Apr96. PUB 1Apr96;
TX 4-319-321. 12Aug96; May96. PUB 1May96;
TX 4-319-321. 12Aug96; Jun96. PUB 1Jun96;

Title: Techniques & solutions
Additional Information: .
Note: Monthly.
Holdings: * acGlobecon Group, Ltd.
TX 4-668-665. 8Dec97; Jan97. (CO corres.) PUB 1Jan97;
TX 4-668-665. 8Dec97; Feb97. (CO corres.) PUB 1Feb97;
TX 4-668-665. 8Dec97; Mar97. (CO corres.) PUB 1Mar97;
TX 4-668-666. 8Dec97; Apr97. (CO corres.) PUB 1Apr97;
TX 4-668-666. 8Dec97; May97. (CO corres.) PUB 1May97;
TX 4-668-666. 8Dec97; Jun97. (CO corres.) PUB 1Jun97;
TX 4-668-667. 8Dec97; Jul97. (CO corres.) PUB 1Jul97;
TX 4-668-667. 8Dec97; Aug97. (CO corres.) PUB 1Aug97;
TX 4-668-667. 8Dec97; Sep97. (CO corres.) PUB 1Sep97;
TX 4-673-998. 14Apr98; Oct97. PUB 1Oct97;
TX 4-673-998. 14Apr98; Nov97. PUB 1Nov97;
TX 4-673-998. 14Apr98; Dec97. PUB 1Dec97;
TX 4-660-547. 14Apr98; Jan98. PUB 1Jan98;
TX 4-660-547. 14Apr98; Feb98. PUB 1Feb98;
TX 4-660-547. 14Apr98; Mar98. PUB 1Mar98;

Title: Techniques & solutions
Additional Information: .
Note: Monthly.
Holdings: * acGlobecon Group, Ltd.
TX 4-761-319. 8Dec97; Jul96. PUB 1Jul96;
TX 4-761-319. 8Dec97; Aug96. PUB 1Aug96;
TX 4-761-319. 8Dec97; Sep96. PUB 1Sep96;
TX 4-761-320. 8Dec97; Oct96. PUB 1Oct96;
TX 4-761-320. 8Dec97; Nov96. PUB 1Nov96;
TX 4-761-320. 8Dec97; Dec96. PUB 1Dec96;
TX 4-711-238. 23Jun98; Apr98. PUB 1Apr98;

TX 4-711-238. 23Jun98; May98. PUB 1May98;
TX 4-711-238. 23Jun98; Jun98. PUB 1Jun98;

Title: Techniques & solutions
Additional Information: .
Note: Monthly.
Holdings: * acGlobecon Group, Ltd.
TX 4-911-468. 4Dec98; Jul98. PUB 1Jul98;
TX 4-911-468. 4Dec98; Aug98. PUB 1Aug98;
TX 4-911-468. 4Dec98; Sep98. PUB 1Sep98;
TX 4-911-460. 12Feb99; Oct98. PUB 1Oct98;
TX 4-911-460. 12Feb99; Nov98. PUB 1Nov98;
TX 4-911-460. 12Feb99; Dec98. PUB 1Dec98;
TX 4-940-912. 26Jul99; Apr99. PUB 1Apr99;
TX 4-940-912. 26Jul99; May99. PUB 1May99;
TX 4-940-912. 26Jul99; Jun99. PUB 1Jun99;

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